

# A general symmetry-preserving observer for aided attitude heading reference systems

Philippe Martin, Erwan Salaün

▶ **To cite this version:**

Philippe Martin, Erwan Salaün. A general symmetry-preserving observer for aided attitude heading reference systems. 47th IEEE Conference on Decision and Control, Dec 2008, Cancun, Mexico. pp.2294 - 2301, 10.1109/CDC.2008.4739372 . hal-00495853

**HAL Id: hal-00495853**

**https://minesparis-psl.hal.science/hal-00495853**

Submitted on 29 Jun 2010

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Proceedings of the
47th IEEE Conference on Decision and Control
Cancun, Mexico, Dec. 9-11, 2008



# A General Symmetry-Preserving Observer for Aided Attitude Heading Reference Systems

Philippe Martin and Erwan Salaün

*Abstract*— **We generalize several recent works on nonlinear observers for aided attitude heading reference systems: we propose a symmetry-preserving nonlinear observer which merges the most common measurements available on an aircraft (altitude, Earth-fixed and body-fixed velocity, inertial and magnetic sensors). It can be seen as an alternative to the Extended Kalman Filter, but easier to tune and computationally much more economic. Moreover it has by design a nice geometrical structure appealing from an engineering viewpoint. We illustrate its good performance on simulation and experimental results.**

## I. Introduction

Aircraft, especially Unmanned Aerial Vehicles (UAV), commonly need to know their orientation and velocity to be operated, whether manually or with computer assistance. When cost or weight is an issue, using very accurate inertial sensors for "true" (i.e. based on the Schuler effect due to a non-flat rotating Earth) inertial navigation is excluded. Instead, low-cost systems –sometimes called aided Attitude Heading Reference Systems (AHRS)– rely on light and cheap "strapdown" gyroscopes, accelerometers and magnetometers "aided" by position and velocity sensors (for instance the velocity vector is given in body-fixed coordinates by an air-data or Doppler radar system, the position/velocity vectors in Earth-fixed coordinates by a GPS engine, the altitude by a barometric altimeter). The various measurements are then "merged" according to the motion equations of the aircraft assuming a flat non-rotating Earth, usually with a linear complementary filter or an Extended Kalman Filter (EKF). For more details about avionics, various inertial navigation systems and sensor fusion, see for instance the books [4], [10], [7], [5], [6] and the references therein.

While the EKF is a general method capable of good performance when properly tuned, it suffers several drawbacks: it is not easy to choose the numerous parameters; it is computationally expensive, which is a problem in low-cost embedded systems; it is usually difficult to prove the convergence, even at first-order, and the designer has to rely on extensive simulations. Moreover on the aided attitude heading problem, suitable modifications must be made to make it work with quaternions [11], [9].

An alternative route is to use an ad hoc nonlinear observer as proposed in [19], [13], [8], [14], [12], [1], [16], [17], [15]. In the absence of a general method the main difficulty is of course to find such an observer. In this paper we use the rich geometric structure of the flying rigid body problem to derive an observer by the method developed in [3], building up on the preliminary work [2]. We propose a general observer structure able to handle the various possible sensors, and which supersedes the previous works [13], [8], [14], [12], [16], [17], [15]. It has by design a nice geometrical structure appealing from an engineering viewpoint; it is easy to tune, computationally very economic, and with guaranteed (at least local) convergence around every trajectory for low velocity flight. Moreover it behaves sensibly in the presence of magnetic disturbances. We illustrate its good performance on simulation and on preliminary experimental comparisons with the commercial Microbotics MIDG II system.

The authors are with the Centre Automatique et Systèmes, École des Mines de Paris, 60 boulevard Saint-Michel, 75272 Paris Cedex 06, FRANCE philippe.martin@ensmp.fr, erwan.salaun@ensmp.fr

## II. Physical equations and measurements

### A. Motion equations

The motion of a flying rigid body (assuming the Earth is flat and defines an inertial frame) is described by

$$\begin{aligned}\dot{q} &= \frac{1}{2} q * \omega \\ \dot{v} &= v \times \omega + q^{-1} * A * q + a \\ \dot{h} &= \langle q * v * q^{-1}, e_3 \rangle\end{aligned}$$

where

- $q$ is the quaternion representing the orientation of the body-fixed frame with respect to the Earth-fixed frame
- $\omega$ is the instantaneous angular velocity vector
- $v$ is the velocity vector of the center of mass with respect to the Body-fixed frame
- $A = ge_3$ is the (constant) gravity vector in North-East-Down coordinates (the unit vectors $e_1, e_2, e_3$ point respectively North, East, Down)
- $a$ is the specific acceleration vector, i.e. all the non-gravitational forces divided by the body mass.
- $h$ is the altitude of the center of mass

More generally we could use $\dot{x} = q * v * q^{-1}$, where $x$ is the position of the center of mass in the Earth-fixed frame, instead of the last equation.

The first equation describes the kinematics of the body, the second is Newton's force law. It is customary to use quaternions (also called Euler 4-parameters) instead of Euler angles since they provide a global parametrization of the body orientation, and are well-suited for calculations and computer simulations. For more details about this section, see any good textbook on aircraft modeling, for instance [18], and section VIII in [14] for useful formulas used in this paper.

978-1-4244-3124-3/08/$25.00 ©2008 IEEE



Authorized licensed use limited to: ECOLE DES MINES PARIS. Downloaded on March 12, 2009 at 08:43 from IEEE Xplore. Restrictions apply.

### B. Measurements

We have four triaxial sensors plus the pressure sensor, providing thirteen scalar measurements: 3 gyros measure $\omega_m = \omega + \omega_b$, where $\omega_b$ is a constant vector bias; 3 accelerometers measure $a$; 3 magnetometers measure $y_b = q^{-1} * B * q$, where $B = B_1 e_1 + B_3 e_3$ is the Earth magnetic field in NED coordinates; the velocity vector $y_v = v$ in the body-fixed frame is provided by some air-data or Doppler radar system; the velocity vector $y_V = V$ is provided by the navigation solution of a GPS engine; a barometric sensor provides the altitude measurement $y_h = h$. All these measurements are of course also corrupted by noise.

It is possible to include more parameters to model the sensors imperfections, see [16], [17], [15]. For simplicity we have considered here only gyros biases.

### C. The considered system

To design our observers we therefore consider the system

$$\dot{q} = \frac{1}{2} q * (\omega_m - \omega_b) \tag{1}$$
$$\dot{v} = v \times (\omega_m - \omega_b) + q^{-1} * A * q + a \tag{2}$$
$$\dot{h} = \langle q * v * q^{-1}, e_3 \rangle \tag{3}$$
$$\dot{\omega}_b = 0 \tag{4}$$

where $\omega_m$ and $a$ are seen as known inputs, together with the output

$$\begin{pmatrix} y_v \\ y_V \\ y_B \\ y_h \end{pmatrix} = \begin{pmatrix} v \\ q * v * q^{-1} \\ q^{-1} * B * q \\ h \end{pmatrix}. \tag{5}$$

## III. Construction of the observer

### A. Invariance of the system equations

There is no general method for designing a nonlinear observer for a given system. When the system has a rich geometric structure, namely invariance with respect to symmetry group, the ideas in [3] provide a constructive method to this problem and simplify the analysis of the error system.

Several translational and rotational groups have been considered for the aided attitude estimation problem [13], [8], [14], [12], [16], [17], [15]. Nevertheless as soon as we also consider position/velocity measurements in both the Earth-fixed and body-fixed frames translations acting on the velocities must be ruled out, see the discussion in IV. We therefore consider the following transformation group generated by constant rotations and translation in the body-fixed and Earth-fixed frames

$$\varphi_{(p_0,q_0,h_0,\omega_0)} \begin{pmatrix} q \\ v \\ h \\ \omega_b \end{pmatrix} = \begin{pmatrix} p_0 * q * q_0 \\ q_0^{-1} * v * q_0 \\ h + h_0 \\ q_0^{-1} * \omega_b * q_0 + \omega_0 \end{pmatrix} = \begin{pmatrix} \tilde{q} \\ \tilde{v} \\ \tilde{h} \\ \tilde{\omega}_b \end{pmatrix}$$
$$\psi_{(p_0,q_0,h_0,\omega_0)} \begin{pmatrix} \omega_m \\ a \\ A \\ B \end{pmatrix} = \begin{pmatrix} q_0^{-1} * \omega_m * q_0 + \omega_0 \\ q_0^{-1} * a * q_0 \\ p_0 * A * p_0^{-1} \\ p_0 * B * p_0^{-1} \end{pmatrix} = \begin{pmatrix} \tilde{\omega}_m \\ \tilde{a} \\ \tilde{A} \\ \tilde{B} \end{pmatrix}$$
$$\rho_{(p_0,q_0,h_0,\omega_0)} \begin{pmatrix} y_v \\ y_V \\ y_B \\ y_h \end{pmatrix} = \begin{pmatrix} q_0^{-1} * y_v * q_0 \\ p_0 * y_V * p_0^{-1} \\ q_0^{-1} * y_B * q_0 \\ y_h + h_0 \end{pmatrix} = \begin{pmatrix} \tilde{y}_v \\ \tilde{y}_V \\ \tilde{y}_B \\ \tilde{y}_h \end{pmatrix}.$$

There are $3+3+1+3 = 10$ parameters: the two unit quaternions $p_0$ and $q_0$, the scalar $h_0$ and the $\mathbb{R}^3$-vector $\omega_0$. The group law $\diamond$ is given by

$$\begin{pmatrix} p_1 \\ q_1 \\ h_1 \\ \omega_1 \end{pmatrix} \diamond \begin{pmatrix} p_0 \\ q_0 \\ h_0 \\ \omega_0 \end{pmatrix} = \begin{pmatrix} p_0 * p_1 \\ q_0 * q_1 \\ h_0 + h_1 \\ q_1^{-1} * \omega_0 * q_1 + \omega_1 \end{pmatrix}.$$

The system (1)–(4) is of course invariant by the transformation group since

$$\begin{aligned}
\dot{\tilde{q}} &= p_0 * \dot{q} * q_0 = p_0 * (\frac{1}{2} q * \omega) * q_0 \\
&= \frac{1}{2} (p_0 * q * q_0) * (q_0^{-1} * \omega * q_0) = \frac{1}{2} \tilde{q} * \tilde{\omega} \\
\dot{\tilde{v}} &= q_0^{-1} * \dot{v} * q_0 \\
&= (q_0^{-1} * v * q_0) \times (q_0^{-1} * \omega * q_0) + q_0^{-1} * a * q_0 \\
&\quad + (p_0 * q * q_0)^{-1} * (p_0 * A * p_0^{-1}) * (p_0 * q * q_0) \\
&= \tilde{v} \times \tilde{\omega} + \tilde{q}^{-1} * \tilde{A} * \tilde{q} + \tilde{a} \\
\dot{\tilde{h}} &= \dot{h} = \langle q * v * q^{-1}, e_3 \rangle = \langle \tilde{q} * \tilde{v} * \tilde{q}^{-1}, \tilde{e}_3 \rangle \\
\dot{\tilde{\omega}}_b &= q_0^{-1} * \dot{\omega}_b * q_0 = 0,
\end{aligned}$$

whereas the output (5) is compatible since

$$\begin{pmatrix} \tilde{v} \\ \tilde{q} * \tilde{v} * \tilde{q}^{-1} \\ \tilde{q}^{-1} * \tilde{B} * \tilde{q} \\ \tilde{h} \end{pmatrix} = \rho_{(p_0,q_0,h_0,\omega_0)} \begin{pmatrix} v \\ q * v * q^{-1} \\ q^{-1} * B * q \\ h \end{pmatrix}.$$

### B. Construction of the general invariant pre-observer

We solve for $(p_0, q_0, h_0, \omega_0)$ the normalization equations

$$\begin{aligned}
p_0 * q * q_0 &= 1 \\
p_0 * e_i * p_0^{-1} &= \tilde{e}_i \quad \text{with} \quad i = 1,2,3 \\
h + h_0 &= 0 \\
q_0^{-1} * \omega_b * q_0 + \omega_0 &= 0
\end{aligned} \tag{6}$$

where $(\tilde{e}_1, \tilde{e}_2, \tilde{e}_3)$ defines a new orthonormal frame. The moving frame $\gamma(q, h, \omega_b, e_1, e_2, e_3)$ is then defined by

$$\begin{aligned}
q_0 &= q^{-1} * p_0^{-1} \\
h_0 &= -h \\
\omega_0 &= -p_0 * q * \omega_b * q^{-1} * p_0^{-1}
\end{aligned}$$



Authorized licensed use limited to: ECOLE DES MINES PARIS. Downloaded on March 12, 2009 at 08:43 from IEEE Xplore. Restrictions apply.

where $p_0$ represents the rotation between the two frames $(e_1,e_2,e_3)$ and $(\tilde{e}_1,\tilde{e}_2,\tilde{e}_3)$. We slightly generalize the construction in [3] since we normalize not only with respect to $\varphi$ but also with respect to $\psi$.

We can then find the 10 scalar invariant errors which correspond to the projections of the output error

$$\rho_{\gamma(\hat{q},\hat{h},\hat{\omega}_b,e_1,e_2,e_3)}\begin{pmatrix}\hat{y}_v\\ \hat{y}_V\\ \hat{y}_B\end{pmatrix}-\rho_{\gamma(\hat{q},\hat{h},\hat{\omega}_b,e_1,e_2,e_3)}\begin{pmatrix}y_v\\ y_V\\ y_B\end{pmatrix}$$

on the new frame $(\tilde{e}_1,\tilde{e}_2,\tilde{e}_3)$ and to the output error for $y_h$ which is directly invariant. We get

$$\begin{aligned}E_{vi}&=\langle\hat{q}*(\hat{y}_v-y_v)*\hat{q}^{-1},e_i\rangle\\ E_{Vi}&=\langle\hat{y}_V-y_V,e_i\rangle\\ E_{Bi}&=\langle B-\hat{q}*y_B*\hat{q}^{-1},e_i\rangle\\ E_h&=\hat{h}-h,\end{aligned}$$

where $i=1,2,3$. We detail how to get $E_{vi}$:

$$\begin{aligned}&\langle\rho_{\gamma(\hat{q},\hat{h},\hat{\omega}_b,e_1,e_2,e_3)}\left(\hat{y}_v\right)-\rho_{\gamma(\hat{q},\hat{h},\hat{\omega}_b,e_1,e_2,e_3)}\left(y_v\right),\tilde{e}_i\rangle\\ &\quad=\langle p_0*\hat{q}*\hat{y}_v*\hat{q}^{-1}*p_0^{-1}-p_0*\hat{q}*y_v*\hat{q}^{-1}*p_0^{-1},\tilde{e}_i\rangle\\ &\quad=\langle\hat{q}*\hat{y}_v*\hat{q}^{-1}-\hat{q}*y_v*\hat{q}^{-1},p_0^{-1}*\tilde{e}_i*p_0\rangle\\ &\quad=\langle\hat{q}*(\hat{y}_v-y_v)*\hat{q}^{-1},e_i\rangle.\end{aligned}$$

We get also the 9 scalar complete invariants which correspond to the projections of

$$\phi_{\gamma(\hat{q},\hat{v},\hat{\omega}_b,e_1,e_2,e_3)}\left(\hat{v}\right)\quad\text{and}\quad\psi_{\gamma(\hat{q},\hat{v},\hat{\omega}_b,e_1,e_2,e_3)}\begin{pmatrix}\omega_m\\ a\end{pmatrix}$$

on the new frame $(\tilde{e}_1,\tilde{e}_2,\tilde{e}_3)$. We find

$$\begin{aligned}I_{\hat{v}i}&=\langle\hat{q}*\hat{v}*\hat{q}^{-1},e_i\rangle\\ I_{\omega_m i}&=\langle\hat{q}*(\omega_m-\hat{\omega}_b)*\hat{q}^{-1},e_i\rangle\\ I_{ai}&=\langle\hat{q}*a*\hat{q}^{-1},e_i\rangle\qquad\text{where}\qquad i=1,2,3.\end{aligned}$$

Notice that $I_{\hat{v}i}$, $I_{\omega_m i}$, $I_{ai}$, $E_{vi}$, $E_{Vi}$, $E_{Bi}$'s and $E_h$ are functions of the estimates and the measurements. Hence they are known quantities which can be used in the construction of the preobserver. It is straightforward to check they are indeed invariant. For instance,

$$\begin{aligned}&\langle\hat{q}*\hat{v}*\hat{q}^{-1},e_i\rangle\\ &\quad=\langle p_0*\hat{q}*\hat{v}*\hat{q}^{-1}*p_0^{-1},p_0*e_i*p_0^{-1}\rangle\\ &\quad=\langle(p_0*\hat{q}*q_0)*(q_0^{-1}*\hat{v}*q_0)*(p_0*\hat{q}*q_0)^{-1},\tilde{e}_i\rangle.\end{aligned}$$

To find invariant vector fields, we solve for $w(q,v,h,\omega_b)$ the 10 vector equations

$$\begin{aligned}&\left[D\varphi_{\gamma(q,v,\omega_b,e_1,e_2,e_3)}\begin{pmatrix}q\\ v\\ h\\ \omega_b\end{pmatrix}\right]\cdot w(q,v,h,\omega_b)\\ &\quad=\begin{pmatrix}\tilde{e}_i\\ 0\\ 0\\ 0\end{pmatrix},\begin{pmatrix}0\\ \tilde{e}_i\\ 0\\ 0\end{pmatrix},\begin{pmatrix}0\\ 0\\ e_7\\ 0\end{pmatrix},\begin{pmatrix}0\\ 0\\ 0\\ \tilde{e}_i\end{pmatrix}\qquad i=1,2,3.\end{aligned}$$

Since

$$\left[D\varphi_{(p_0,q_0,h_0,\omega_0)}\begin{pmatrix}q\\ v\\ h\\ \omega_b\end{pmatrix}\right]\cdot\begin{pmatrix}\delta q\\ \delta v\\ \delta h\\ \delta\omega_b\end{pmatrix}=\begin{pmatrix}p_0*\delta q*q_0\\ q_0^{-1}*\delta v*q_0\\ \delta h\\ q_0^{-1}*\delta\omega_b*q_0\end{pmatrix},$$

this yields the 10 independent invariant vector fields $(i=1,2,3)$

$$\begin{pmatrix}e_i*q\\ 0\\ 0\\ 0\end{pmatrix},\begin{pmatrix}0\\ q^{-1}*e_i*q\\ 0\\ 0\end{pmatrix},\begin{pmatrix}0\\ 0\\ e_7\\ 0\end{pmatrix},\begin{pmatrix}0\\ 0\\ 0\\ q^{-1}*e_i*q\end{pmatrix}.$$

Indeed for instance the equations $p_0*\delta q*q_0=\tilde{e}_i$ gave us

$$\delta q=p_0^{-1}*\tilde{e}_i*q_0^{-1}=(p_0^{-1}*\tilde{e}_i*p_0)*(q_0*p_0)^{-1}=e_i*q.$$

It is easy to check that these vector fields are invariant. The general invariant preobserver then reads

$$\begin{aligned}\dot{\hat{q}}&=\frac{1}{2}\hat{q}*(\omega_m-\hat{\omega}_b)\\ &\quad+\sum_{i=1}^{3}\Big(\sum_{j=1}^{3}(l_{vij}E_{vj}+l_{Vij}E_{Vj}+l_{Bij}E_{Bj})+l_{hi}E_h\Big)e_i*\hat{q}\\ \dot{\hat{v}}&=\hat{v}\times(\omega_m-\hat{\omega}_b)+\hat{q}^{-1}*A*\hat{q}+a+\hat{q}^{-1}*\Big(\sum_{i=1}^{3}\Big(m_{hi}E_h\\ &\quad+\sum_{j=1}^{3}(m_{vij}E_{vj}+m_{Vij}E_{Vj}+m_{Bij}E_{Bj})\Big)e_i\Big)*\hat{q}\\ \dot{\hat{h}}&=\langle\hat{q}*\hat{v}*\hat{q}^{-1},e_3\rangle+\sum_{j=1}^{3}(n_{vj}E_{vj}+n_{Vj}E_{Vj}+n_{Bj}E_{Bj})+n_hE_h\\ \dot{\hat{\omega}}_b&=\hat{q}^{-1}*\Big(\sum_{i=1}^{3}\Big(\sum_{j=1}^{3}(o_{vij}E_{vj}+o_{Vij}E_{Vj}+o_{Bij}E_{Bj})\\ &\quad+o_{hi}E_h\Big)e_i\Big)*\hat{q},\end{aligned}$$

where the $l_{vij}$, $l_{Vij}$, $l_{Bij}$, $l_{hi}$, $m_{vij}$, $m_{Vij}$, $m_{Bij}$, $m_{hi}$, $n_{vj}$, $n_{Vj}$, $n_{Bj}$, $o_{vij}$, $o_{Vij}$, $o_{Bij}$, $o_{hi}$'s and $n_h$ are arbitrary scalars which possibly depend on $E_{vi}$, $E_{Vi}$, $E_{Bi}$, $E_h$, $I_{\hat{v}i}$, $I_{\omega_m i}$ and $I_{ai}$'s. Noticing

$$\sum_{i=1}^{3}\Big(\sum_{j=1}^{3}(l_{vij}E_{vj})\Big)e_i=L_vE_v$$

where $E_v=\hat{q}*(\hat{v}-v)*\hat{q}^{-1}$ and $L_v$ is the $3\times3$ matrix whose coefficients are the $l_{vij}$'s, and defining $E_V$, $E_B$, $L_V$, $L_B$, $L_h$, $M_v$, $M_V$, $M_B$, $M_h$, $N_v$, $N_V$, $N_B$, $N_h$, $O_v$, $O_V$, $O_B$ and $O_h$ in the same way, the correction terms can be rewritten with the matrices $E,L,M,N$ and $O$ such as $E=(E_v\ E_V\ E_B\ E_h)^T$ and

$$L_vE_v+L_VE_V+L_BE_B+L_hE_h=LE,$$

and the same notation for $M,N$ and $O$.

Then we can rewrite the preobserver as

$$\dot{\hat{q}}=\frac{1}{2}\hat{q}*(\omega_m-\omega_b)+(LE)*\hat{q}\tag{7}$$

$$\dot{\hat{v}}=\hat{v}\times(\omega_m-\omega_b)+\hat{q}^{-1}*A*\hat{q}+a+\hat{q}^{-1}*(ME)*\hat{q}\tag{8}$$

$$\dot{\hat{h}}=\langle\hat{q}*\hat{v}*\hat{q}^{-1},e_3\rangle+(NE)\tag{9}$$

$$\dot{\hat{\omega}}_b=\hat{q}^{-1}*(OE)*\hat{q}.\tag{10}$$



Authorized licensed use limited to: ECOLE DES MINES PARIS. Downloaded on March 12, 2009 at 08:43 from IEEE Xplore. Restrictions apply.

The preobserver is indeed invariant by considering the projection of the output errors $E_v$, $E_V$ and $E_B$ on the frame $(e_1,e_2,e_3)$. As a by-product of its geometric structure, the preobserver automatically has a desirable feature: the norm of $\hat{q}$ is left unchanged by (7), since $LE$ is a vector of $\mathbb{R}^3$ (see section VIII in [14]).

### C. Error equations

The state error is given by

$$\begin{pmatrix}\eta\\ \nu\\ \lambda\\ \beta\end{pmatrix} = \varphi_{\gamma(q,v,\omega_b,e_1,e_2,e_3)}\begin{pmatrix}\hat{q}\\ \hat{v}\\ \hat{h}\\ \hat{\omega}_b\end{pmatrix} - \varphi_{\gamma(q,v,\omega_b,e_1,e_2,e_3)}\begin{pmatrix}q\\ v\\ h\\ \omega_b\end{pmatrix} = \begin{pmatrix}\hat{q}*q^{-1}-1\\ q*(\hat{v}-v)*q^{-1}\\ \hat{h}-h\\ q*(\hat{\omega}_b-\omega_b)*q^{-1}\end{pmatrix}.$$

It is in fact more natural –though completely equivalent– to take $\eta=\hat{q}*q^{-1}$ (rather than $\eta=\hat{q}*q^{-1}-1$), so that $\eta(x,x)=1$, the unit element of the group of quaternions. As we did above it can be easily checked that $\lambda$ and the projections of $\eta$, $\nu$ and $\beta$ on the frame $(e_1,e_2,e_3)$ are invariant. Hence for $(i,j)=1,2,3$,

$$\begin{aligned}\dot{\overline{\langle \eta*e_i*\eta^{-1},e_j\rangle}} &= \langle \dot{\eta}*e_i*\eta^{-1}-\eta*e_i*\eta^{-1}*\dot{\eta}*\eta^{-1},e_j\rangle\\ &= 2\langle(-\frac{1}{2}\eta*\beta*\eta^{-1})\times(\eta*e_i*\eta^{-1}),e_j\rangle\\ &\quad +2\langle(LE)\times(\eta*e_i*\eta^{-1}),e_j\rangle\\ \langle\dot{\nu},e_i\rangle &= \langle(\eta^{-1}*I_{\hat{v}}*\eta)\times\beta+\eta^{-1}*A*\eta-A,e_i\rangle\\ &\quad +\langle\eta^{-1}*(ME)*\eta,e_i\rangle\\ \dot{\lambda} &= \dot{\hat{h}}-\dot{h}\\ &= \langle I_{\hat{v}}-\eta^{-1}*I_{\hat{v}}*\eta-\eta^{-1}*\nu*\eta,e_3\rangle+(NE)\\ \langle\dot{\beta},e_i\rangle &= \langle(\eta^{-1}*I_{\omega_m}*\eta)\times\beta+\eta^{-1}*(OE)*\eta,e_i\rangle.\end{aligned}$$

Since we can write

$$E_v=\eta*\nu*\eta^{-1} \qquad E_V=I_{\hat{v}}-\eta^{-1}*I_{\hat{v}}*\eta+\nu$$
$$E_B=B-\eta*B*\eta^{-1} \qquad E_h=\lambda,$$

we find as expected that the error system

$$\dot{\overline{\eta*e_i*\eta^{-1}}} = 2(-\frac{1}{2}\eta*\beta*\eta^{-1})\times(\eta*e_i*\eta^{-1}) + 2(LE)\times(\eta*e_i*\eta^{-1}) \quad (11)$$

$$\dot{\nu} = (\eta^{-1}*I_{\hat{v}}*\eta)\times\beta+\eta^{-1}*A*\eta-A + \eta^{-1}*(ME)*\eta \quad (12)$$

$$\dot{\lambda} = \langle I_{\hat{v}}-\eta^{-1}*I_{\hat{v}}*\eta-\eta^{-1}*\nu*\eta,e_3\rangle+(NE) \quad (13)$$

$$\dot{\beta} = (\eta^{-1}*I_{\omega_m}*\eta)\times\beta+\eta^{-1}*(OE)*\eta \quad (14)$$

depends only on the invariant state error $(\eta,\nu,\lambda,\beta)$ and the "free" known invariants $I_{\hat{v}}$ and $I_{\omega_m}$, but not on the trajectory of the observed system (1)–(4). The error system (11)–(14) is invariant by considering the projection of the equations (11),(12) and (14) on the frame $(e_1,e_2,e_3)$.

The linearized error system around $(\overline{\eta},\overline{\nu},\overline{\lambda},\overline{\beta})=(1,0,0,0)$, i.e. the estimated state equals the actual state, is given by

$$\delta\dot{\eta} = -\frac{1}{2}\delta\beta+(L\delta E) \quad (15)$$

$$\delta\dot{\nu} = I_{\hat{v}}\times\delta\beta+2A\times\delta\eta+(M\delta E) \quad (16)$$

$$\delta\dot{\lambda} = \langle -I_{\hat{v}}\times\delta\eta-\delta\nu,e_3\rangle+(N\delta E) \quad (17)$$

$$\delta\dot{\beta} = I_{\omega_m}\times\beta+(O\delta E), \quad (18)$$

where

$$\delta E_v=\delta\nu \qquad \delta E_V=\delta\nu-2I_{\hat{v}}\times\delta\nu$$
$$\delta E_B=2B\times\delta\eta \qquad \delta E_h=\delta\lambda.$$

We notice that the normalization equation (6) let us to consider the velocity error $\nu$ in the Earth-fixed frame. Instead, choosing $q_0^{-1}*e_i*q_0=\tilde{e}_i$ would lead us a velocity error $\tilde{\nu}=\hat{v}-v$ in the body-fixed frame which seems more "natural" since the equation (2) is written in body-fixed coordinates. But in this case, the output error $\delta E_B$ would become $\delta\tilde{E}_B=2(\hat{q}^{-1}*B*\hat{q})\times\delta\nu$; so the error system, and its convergence behavior, would depend on the trajectory $\hat{q}(t)$.

## IV. Position with respect to previous works

The invariant observer (7)–(10) supersedes several previous works, which are detailed in this section.

### A. No velocity measurements: [8]-[14]-[16]

In [8]-[14]-[16] only inertial and magnetic sensors are used. So the linear acceleration $\dot{v}-v\times\omega$ is supposed small enough to approximate the accelerometers measurements by $y_a=-q^{-1}*A*q$. We recover the observer described in [8]-[14]-[16] considering only (1) and (4) as the physical system and $y_a$ and $y_b$ as the output.

### B. Ideal measurements: [1]

The nonlinear complementary filter presented in [1] use ideal measurements: pose estimation (computationally demanding) and no biases.

### C. Transformation incompatible with body-fixed velocity: [17]

In [17] only the Earth-fixed measurements are used. The constructed observer can not be adapted to the body-fixed measurements since with the chosen transformation group the output $y_v$ is not compatible.

### D. No body-fixed velocity: [15]

The observer proposed in [15] is written in the Earth-fixed frame because no body-fixed velocity measurements are considered. We recover this observer transposing (8) in the Earth-fixed frame.



Authorized licensed use limited to: ECOLE DES MINES PARIS. Downloaded on March 12, 2009 at 08:43 from IEEE Xplore. Restrictions apply.

*E. Transformation incompatible with Earth-fixed velocity: [3]*

In [3] only the body-fixed measurements are used and no biases are considered. The constructed observer can not be adapted to the body-fixed measurements since the output $y_v$ is not compatible with the chosen transformation group.

*F. Particular case: [2]*

The observer (7)–(10) generalizes the observer presented in [2]. Indeed we recover it taking

$$L_vE_v = -l_1Ev - l_2E_v \times A - l_3\langle E_v, A\rangle A$$
$$M_vE_v = -l_4Ev - l_5E_v \times A - l_6\langle E_v, A\rangle A$$

and considering no bias and no Earth-fixed measurements.

## V. Choice of the observer parameters

Up to now, we have only investigated the structure of the observer. We now must choose the gain matrices $L_v$, $L_V$, $L_B$, $L_h$, $M_v$, $M_V$, $M_B$, $M_h$, $N_v$, $N_V$, $N_B$, $N_h$, $O_v$, $O_V$, $O_B$, $O_h$ to meet the following requirements *locally around any trajectory*:

- at a low velocity "normal" flight, which is common for UAVs in an urban area, i.e. $I_{\hat{v}}$ and $I_{\omega_m}$ are first order terms: the error must converge to zero and its behavior should be easily tunable; the magnetic measurements should not affect the attitude, velocity and altitude estimations, but only the heading
- at a level flight, i.e. $I_{\hat{v}} = V_1e_1 + V_2e_2 + \delta V_3e_3$ where $V_1, V_2$ are constant and $I_{\omega_m}, \delta V_3$ are first order terms, the behavior of the roll angle error towards the direction of $I_{\hat{v}}$, which is the most important estimation for a flight control, should not be affected by $V_1$,$V_2$,$\delta V_3$ and any magnetic disturbance.

Therefore we choose

$$L_vE_v = l_vA \times E_v \qquad L_VE_V = l_VA \times E_V$$
$$L_BE_B = l_B\langle B \times E_B, A\rangle A$$
$$M_vE_v = -m_vE_v \qquad M_VE_V = -m_VE_V$$
$$N_hE_h = -n_hE_h$$
$$O_vE_v = -o_vA \times E_v \qquad O_VE_V = -o_VA \times E_V$$
$$O_BE_B = -o_B\langle B \times E_B, A\rangle A$$

with $(l_v, l_V, l_B, m_v, m_V, n_h, o_v, o_V, o_B) > 0$ and the other matrices equal to 0. At a low velocity "normal" flight the error system (15)–(18) splits into three decoupled subsystems and two cascaded subsystems:

- the longitudinal subsystem

$$\begin{pmatrix}\delta\dot{\eta}_2\\ \delta\dot{v}_1\\ \delta\dot{\beta}_2\end{pmatrix} = \begin{pmatrix}0 & g*(l_v+l_V) & -\frac{1}{2}\\ -2g & -(m_v+m_V) & 0\\ 0 & -(o_v+o_V) & 0\end{pmatrix}\begin{pmatrix}\delta\eta_2\\ \delta v_1\\ \delta\beta_2\end{pmatrix}$$

- the lateral subsystem

$$\begin{pmatrix}\delta\dot{\eta}_1\\ \delta\dot{v}_2\\ \delta\dot{\beta}_1\end{pmatrix} = \begin{pmatrix}0 & -g*(l_v+l_V) & -\frac{1}{2}\\ 2g & -(m_v+m_V) & 0\\ 0 & o_v+o_V & 0\end{pmatrix}\begin{pmatrix}\delta\eta_1\\ \delta v_2\\ \delta\beta_1\end{pmatrix}$$

- the vertical subsystem

$$\delta\dot{v}_3 = -(m_v+m_V)\delta v_3$$

- the heading subsystem

$$\begin{pmatrix}\delta\dot{\eta}_3\\ \delta\dot{\beta}_3\end{pmatrix} = \begin{pmatrix}-2gB_1^2l_B & -\frac{1}{2}\\ 2gB_1^2o_B & 0\end{pmatrix}\begin{pmatrix}\delta\eta_3\\ \delta\beta_3\end{pmatrix} + \begin{pmatrix}2gB_1B_3l_B\\ -2gB_1B_3o_B\end{pmatrix}\delta\eta_1$$

- the altitude subsystem

$$\delta\dot{\lambda} = -n_h\delta\lambda - \delta v_3.$$

Thanks to this decoupled structure, the tuning of the gains $l_v, l_V, l_B, m_v, m_V, n_h, o_v, o_V, o_B$ is straightforward. Obviously the lateral, longitudinal, vertical and altitude subsystems do not depend on the magnetic measurements, so will not be affected if the magnetic field is perturbed. We do not detail in this article how the choice of matrices meet the preceding requirements during a level flight. We only illustrate it through simulation and experimental results.

## VI. Simulation results

We first illustrate on simulation the behavior of the invariant observer

$$\dot{\hat{q}} = \frac{1}{2}\hat{q} * (\omega_m - \hat{\omega}_b) + (LE) * \hat{q} + \alpha(1 - \|\hat{q}\|^2)\hat{q}$$
$$\dot{\hat{v}} = \hat{v} \times (\omega_m - \hat{\omega}_b) + \hat{q}^{-1} * A * \hat{q} + a + \hat{q}^{-1} * (ME) * \hat{q}$$
$$\dot{\hat{h}} = \langle \hat{q} * \hat{v} * \hat{q}^{-1}, e_3\rangle + (NE)$$
$$\dot{\hat{\omega}}_b = \hat{q}^{-1} * (OE) * \hat{q}$$

with the choice of gain matrices described in section V. The added term $\alpha(1 - \|\hat{q}\|^2)\hat{q}$ is a well-known numerical trick to keep $\|\hat{q}\| = 1$. Notice this term is invariant.

We choose here time constants around 20s by taking $l_v = l_V = 1e-2$, $l_B = 5.2e-3$, $m_v = m_V = 2.4$, $n_h = 1$, $o_v = o_V = 5.3e-3$, $o_B = 2.8e-3$ and $\alpha = 1$. The system follows the trajectory defined by

$$a = \begin{pmatrix}.4g * \sin(t)\\ .4g * \sin(.5t + \pi/4)\\ -g - .4g\cos(.5t)\end{pmatrix}$$
$$\omega = \begin{pmatrix}.5\sin(.5t)\\ .9\sin(.3t)\\ -.5\sin(.25t)\end{pmatrix} \quad \omega_b = \begin{pmatrix}.01\\ .008\\ -.01\end{pmatrix},$$

which is representative of a small UAV flight. We see on the Fig. 1–4 the results of the following experiment:

- for $t < 40s$ the observer converges well. Though we have no proof of convergence but local, the domain of attraction seems to be large enough since the states are initialized far from their true values and the system moves quite fast
- at $t = 40s$ the "GPS correction terms" are switched off, i.e. the gains $l_V$, $m_V$ and $o_V$ are set to 0. The observer still behaves well
- at $t = 60s$ the magnetic field is changed from $B = (1\ 0\ 1)^T$ to $B = (0.5\ \ -0.8\ \ 0.7)^T$. As expected,



Authorized licensed use limited to: ECOLE DES MINES PARIS. Downloaded on March 12, 2009 at 08:43 from IEEE Xplore. Restrictions apply.

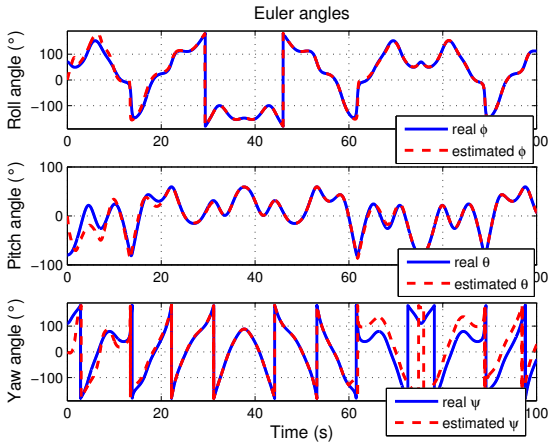



Fig. 1. Estimated Euler angles (simulation)

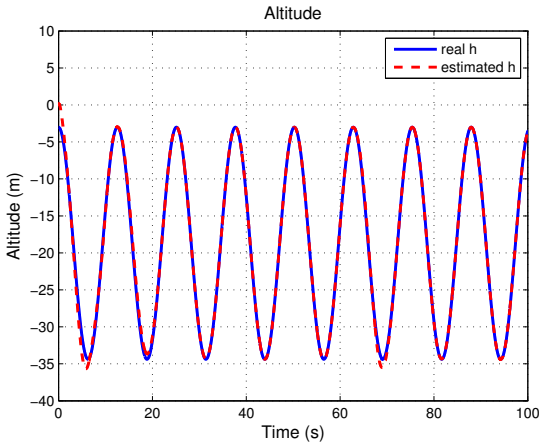



Fig. 3. Estimated altitude (simulation)

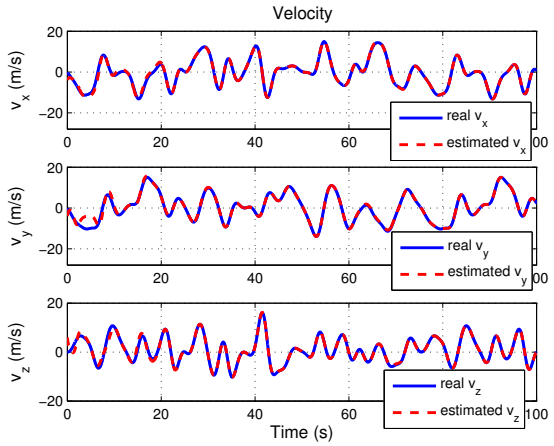



Fig. 2. Estimated velocity (simulation)

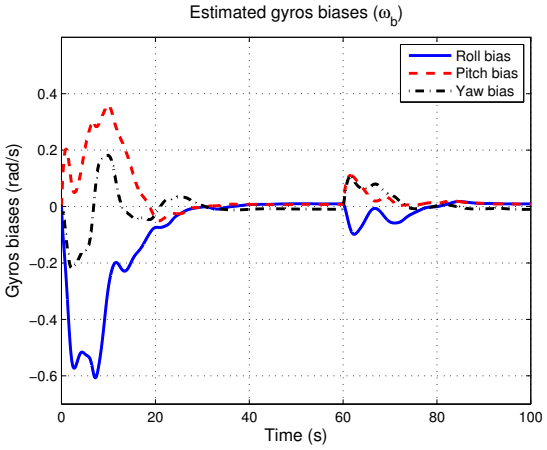



Fig. 4. Estimated biases (simulation)

only the estimated yaw angle $\psi$ is strongly affected by the magnetic disturbance. Because of the coupling terms $I_{\hat{\nu}}$ and $I_{\omega_m}$, there is some dynamic influence on the other variables.

## VII. Experimental results

We do not have yet measurements from an air-data system, so we will use only Earth-fixed velocity, inertial and magnetic measurements provided by the commercial INS-GPS device MIDG II from Microbotics Inc and altitude measurement given by the barometer module Intersema MS5534B. For each experiment we first save the raw measurements from the MIDG II gyros, acceleros and magnetic sensors (at a 50Hz refresh rate), the velocity provided by the navigation solutions of its GPS engine (at a 5 Hz refresh rate) and the raw measurements from a barometer module (at a 12.5 Hz refresh rate). A microcontroller on a development kit communicate with these devices and send the measurements to a computer via the serial port (see Fig. 5). On Matlab Simulink we feed offline the observer with these data and then compare the estimations of the observer to the estimations given by the MIDG II (computed according to the user manual by some Kalman filter). In order to have similar behaviors and considering the units of the raw measurements provided by the MIDG II, we have chosen $l_V = 2.8e-5$, $l_B = 1.4e-6$, $m_V = 9e-3$, $n_h = 5e-2$, $o_V = 4e-7$, $o_B = 2e-8$ and $\alpha = 1$ and we initialized the altitude measurement to 0 at the beginning of the experiment.

### A. Dynamic behavior (Fig. 6–8)

We wait a few minutes until the biases reach constant values, then move the system in all directions. The observer and the MIDG II give very similar results (Fig. 6–8). To do comparison in the same frame we compare the Earth velocity provided by the MIDG II and $\hat{V} = \hat{q} * \hat{v} * \hat{q}^{-1}$ given by our observer. We can notice that the estimation of $V_z$ given by our observer seems to be closer to the real value than the estimation provided by the MIDG II: we know that we let the system motionless at $t = 42s$, which is coherent with our estimated $\hat{V}_z$.

### B. Influence of magnetic disturbances (Fig. 9–12)

Once the biases have reached constant values, the system is left motionless for $60s$. At $t = 72s$ a magnet is put close to



Authorized licensed use limited to: ECOLE DES MINES PARIS. Downloaded on March 12, 2009 at 08:43 from IEEE Xplore. Restrictions apply.

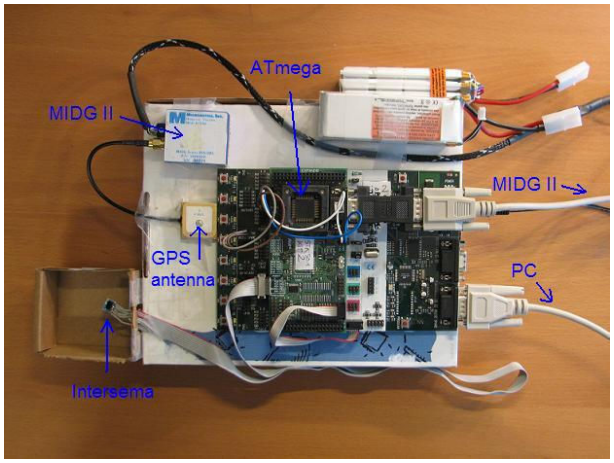



Fig. 5. Experimental protocol

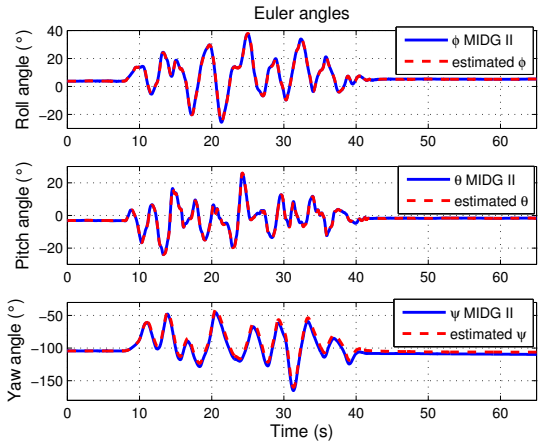



Fig. 6. Estimated Euler angles (experiment)

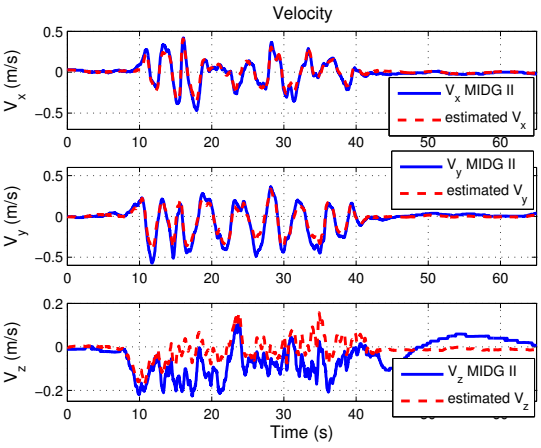



Fig. 7. Estimated velocity (experiment)

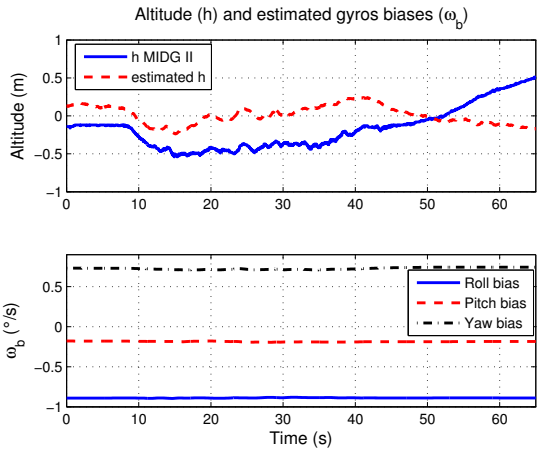



Fig. 8. Estimated altitude and biases (experiment)

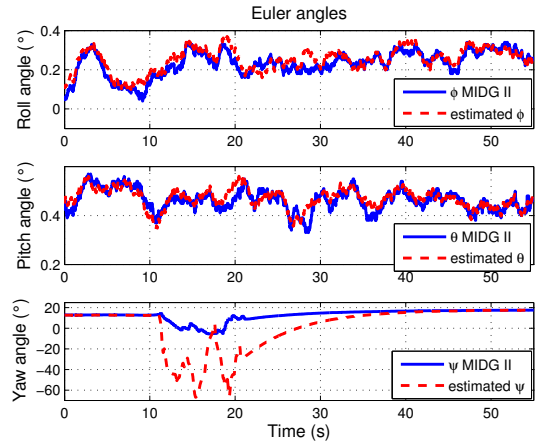



Fig. 9. Influence of magnetic disturbances (experiment)

the sensors for 10$s$. As expected the estimated roll and pitch angles, longitudinal and lateral velocities are not affected by the magnetic disturbance (Fig. 9–10); the MIDG II exhibits a similar behavior. However we notice that the yaw angle estimated by our algorithm is much more affected by the disturbance that the estimation provided by the MIDG II. And thus the estimated vertical velocity and altitude are also perturbed (Fig. 9–11). Indeed for the experiments related to the Fig. 9–11 we used the gain values detailed above, which are constant whatever the norm of the magnetic field is. If the norm of the magnetic measurements change, which means that there is some magnetic disturbance, we would like this to affect the gain values of the magnetic correction terms. For instance if the norm of $y_B$ increases we want to decrease the gains $l_B$ and $o_B$ of the "magnetic correction terms". A first possibility is to consider the gains $l_B$ and $o_B$ divided by $\|y_B\|^{1.5}$, supposing $\|y_B\| \neq 0$. On the Fig. 12 we see that the estimated vertical velocity, altitude and yaw angle are really less disturbed, and are now close to the estimations given by the MIDG II.



Authorized licensed use limited to: ECOLE DES MINES PARIS. Downloaded on March 12, 2009 at 08:43 from IEEE Xplore. Restrictions apply.

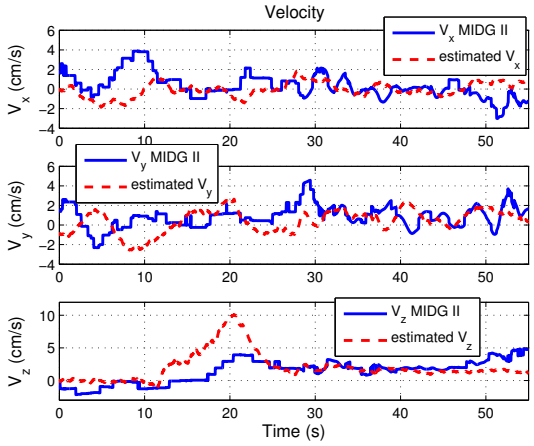



Fig. 10. Influence of magnetic disturbances (experiment)

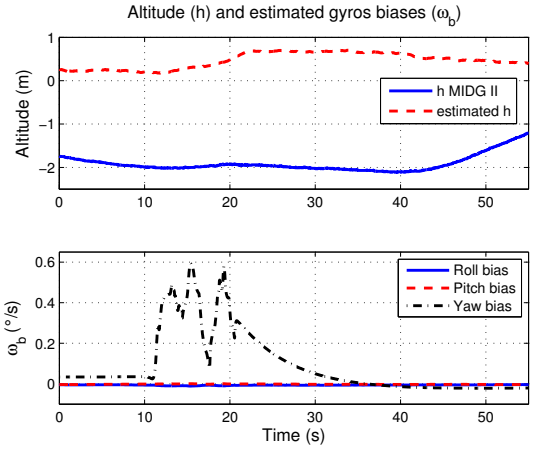



Fig. 11. Influence of magnetic disturbances (experiment)

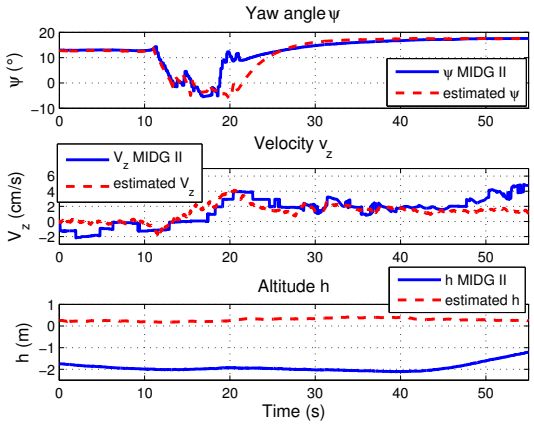



Fig. 12. Influence of magnetic disturbances with adapted gains (experiment)

## VIII. Conclusion and Future Works

In this paper we proposed a general nonlinear observer for aided attitude heading reference systems. It can merge several kinds of measurements from Earth-fixed and body-fixed low-cost sensors. We illustrated its well-behavior and nice properties on simulation and preliminary experimental results. Further work is in progress to implement the observer on a cheap microcontroller, see [16], [15], to demonstrate the computational simplicity of the observer.

## References

[1] G. Baldwin, R. Mahony, J. Trumpf, T. Hamel, and T. Cheviron. Complementary filter design on the special euclidean group SE(3). In *Proc. of the 2007 European Control Conference*, 2007.

[2] S. Bonnabel, Ph. Martin, and P. Rouchon. A non-linear symmetry-preserving observer for velocity-aided inertial navigation. In *Proc. of the 2006 American Control Conference*, pages 2910–2914, 2006.

[3] S. Bonnabel, Ph. Martin, and P. Rouchon. Symmetry-preserving observers. *arxiv.math.OC/0612193*, 2007. Accepted for publication in IEEE Trans. Automat. Control.

[4] R.P.G. Collinson. *Introduction to avionics systems*. Kluwer Academic Publishers, second edition, 2003.

[5] J.A. Farrell and M. Barth. *The global positioning system and inertial navigation*. McGraw-Hill, 1998.

[6] M.S. Grewal and A.P. Andrews. *Kalman filtering : Theory and Practice using MATLAB*. Wiley, second edition, 2001.

[7] M.S. Grewal, L.R. Weill, and A.P. Andrews. *Global positioning systems, inertial navigation, and integration*. Wiley, second edition, 2007.

[8] T. Hamel and R. Mahony. Attitude estimation on $SO(3)$ based on direct inertial measurements. In *Proc. of the 2006 IEEE International Conference on Robotics and Automation*, pages 2170–2175, 2006.

[9] Y.M. Huang, F.R. Chang, and L.S. Wang. The attitude determination algorithm using integrated GPS/INS data. In *Proc. of the* 16*th IFAC World Congress*, 2005.

[10] M. Kayton and W.R. Fried, editors. *Avionics navigation systems*. Wiley, second edition, 1997.

[11] E.J. Lefferts, F.L. Markley, and M.D. Shuster. Kalman filtering for spacecraft attitude. *Journal of Guidance and Control*, 5(5):417–429, 1982.

[12] R. Mahony, T. Hamel, and J-M. Pfimlin. Non-linear complementary filters on the special orthogonal group. *IEEE Trans. Automat. Control*, 2008. To appear.

[13] R. Mahony, T. Hamel, and J.-M. Pflimlin. Complementary filter design on the special orthogonal group $SO(3)$. In *Proc. of the* 44*th IEEE Conf. on Decision and Control*, pages 1477–1484, 2005.

[14] Ph. Martin and E. Salaün. Invariant observers for attitude and heading estimation from low-cost inertial and magnetic sensors. In *Proc. of the* 46*th IEEE Conf. on Decision and Control*, pages 1039–1045.

[15] Ph. Martin and E. Salaün. Design and implementation of a low-cost aided attitude and heading reference system. In *Proc. of the 2008 AIAA Guidance, Navigation, and Control Conference*, 2008. To appear.

[16] Ph. Martin and E. Salaün. Design and implementation of a low-cost attitude and heading nonlinear estimator. In *Proc. of the* 5*th International Conference on Informatics in Control, Automation and Robotics*, 2008. To appear.

[17] Ph. Martin and E. Salaün. An invariant observer for Earth-velocity-aided attitude heading reference systems. In *Proc. of the* 17*th IFAC World Congress*, 2008. To appear.

[18] B.L. Stevens and F.L. Lewis. *Aircraft control and simulation*. Wiley, second edition, 2003.

[19] J. Thienel and R.M. Sanner. A coupled nonlinear spacecraft attitude controller and observer with an unknown constant gyro bias and gyro noise. *IEEE Trans. Automat. Control*, 48(11):2011–2015, 2003.



Authorized licensed use limited to: ECOLE DES MINES PARIS. Downloaded on March 12, 2009 at 08:43 from IEEE Xplore. Restrictions apply.