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Abstract: Linear time-varying systems driven by periodic input signals are ubiquitous in control systems. For various reasons, including disturbance rejection and diagnosis by analysis of the trajectories, estimation of their input signals is often desirable. In the present paper, we illustrate a recently proposed general method to solve such problems by an asymptotic reconstruction of the Fourier expansion of the unknown input signal along with several examples from the automotive engine industry, and with an oscillating water column retrieving wave energy.

Keywords: Observers, Linear Time-Varying systems, Periodic input signals, Automotive applications, wave energy.

1. INTRODUCTION

As is well known, linear time-varying systems driven by periodic input signals are ubiquitous in control systems (see Bittanti and Colaneri (2009)). For various reasons, including disturbance rejection and diagnosis by analysis of the trajectories, estimation of their input signals is often desirable. In the present paper, we present a general method to address such problems along with several examples which serve to see its merits.

In some recent works (see Chauvin et al. (20067)), a strong motivation to aim at reconstructing unknown periodic input signals has raised from automotive engine applications. In this domain of engineering, this periodicity stems from a fundamental property of the engines : at various levels of modelling, automotive engine dynamics can be considered as a system being mechanically coordinated and synchronized by the revolution of the crankshaft. We will show examples stressing the interest of having observers in this context of automotive engine control. Moreover, as will appear, such observer design can be applied to numerous fields of engineering (such as the presented wave energy retrieving system).

We now briefly outline the method we propose. Consider a T_0-periodic input signal denoted w, where T_0 is perfectly known. The case of signals w that could be written as a sum of a *finite* number of harmonics was considered in Chauvin et al. (20067). In this context, a finite-dimensional linear time-varying observer was proposed. We proposed in Chauvin and Petit (2010) an infinite-dimensional observer to reconstruct signals possessing an *infinite* Fourier expansion. This extension provides a simple asymptotic formula that, when truncated, serves as a tuning methodology for finite-dimensional filters. Further, it guarantees global convergence.

This approach is related to several research works in the literature. Online estimation of the frequencies of a signal being the sum of a finite number of sinusoids with unknown magnitudes, frequencies, and phases has been addressed by numerous authors (one can refer to e.g. Hsu et al. (1999); Marino and Tomei (2000); Xia (2002)). However, the problem we address is different. The signal we wish to estimate is not directly measured. It is filtered through a linear time-varying system. The filtered signal is the only available information. Secondly (and very importantly), its period is precisely known. This particularity suggests a dedicated observation technique could be worth developing. Our approach can be considered close to the general class of methods aiming at identifying periodic disturbances in view of cancelling them. The main difficulty lies in determining a simple and mathematically consistent method to tune the gains of the infinite number of adaptation laws. As will be presented, a simple solution is found.

The goal of this paper is to show the application of this method to several examples. In Section 2, we recall the problem statement and the procedure for the observer design. Then, in Section 3, a first example is the inversion of automotive sensor dynamics (see Hammerschmidt and Leteinturier (2004) and Heywood (1988) for more details). This will serve as a tutorial example to illustrate the convergence and the robustness of the observer design. In Section 4, a second automotive example is given. Finally, we present a mechanical system retrieving wave energy in Section 5. We hope that these examples can serve as benchmarks and subject of future studies for the community.

2. STATEMENT OF THE PROBLEM AND PRESENTED OBSERVER DESIGN

We now briefly present the problem under consideration and the solution we propose for it.

Notations

In the following, n and m are strictly positive integers, T_0 is a strictly positive real parameter, $\|\cdots\|_n$ refers to the Euclidean norm of $\mathbb{C}^n$, and $\|\cdots\|_{nm}$ refers to the Euclidean norm of $\mathcal{M}_{n,m}(\mathbb{R})$ the set of $n \times m$ matrices with real entries. The symbol † indicates the Hermitian transpose.

We define

$$\begin{cases} \ell_n^2 \triangleq \{\{x_k\}_{k\in\mathbb{Z}} \in (\mathbb{C}^n)^{\mathbb{Z}} \ / \ \sum_{k\in\mathbb{Z}} \|x_k\|_n^2 < +\infty\} \\ \omega_n^{1,2} \triangleq \{\{x_k\}_{k\in\mathbb{Z}} \in (\mathbb{C}^n)^{\mathbb{Z}} \ / \\ \qquad \sum_{k\in\mathbb{Z}} (1+k^2)\|x_k\|_n^2 < +\infty\} \end{cases}$$

Both ℓ_n^2 and $\omega_n^{1,2}$ are Hilbert spaces with the inner product $< x, y >_{\ell_n^2} = \sum_{k\in\mathbb{Z}} \left\|x_k^\dagger y_k\right\|_n^2$, and $< x, y >_{\omega_n^{1,2}} = \sum_{k\in\mathbb{Z}}(1+k^2)\left\|x_k^\dagger y_k\right\|_n^2$, respectively [1].

We also consider the following functional spaces (Adams, 1975, pp. 23, 60):

$$\begin{cases} L_n^2[0,T_0] \triangleq \{\{[0,\ T_0] \ni t \mapsto x(t) \in \mathbb{R}^n\} \\ \qquad \text{measurable over } [0,\ T_0] \text{ such that} \\ \qquad \int_0^{T_0} \|x(t)\|_n^2\, dt < +\infty\} \\ W_n^{1,2}[0,T_0] \triangleq \{\{[0,\ T_0] \ni t \mapsto x(t) \in \mathbb{R}^n\} \in L_n^2[0,T_0] \\ \qquad \text{such that} \quad Dx \in L_n^2[0,T_0]\}, \end{cases}$$

where Dx is the weak derivative of x.

As will become apparent, the functions considered in this paper have continuous partial derivatives (in the classical sense).

Again, both $L_n^2[0,T_0]$ and $W_n^{1,2}[0,T_0]$ are Hilbert spaces. Moreover, $W_n^{1,2}[0,T_0]$ is a Sobolev space.

We consider the space $\mathbb{E} \triangleq \mathbb{R}^n \times \omega_m^{1,2}$ and note its elements $\mathcal{X} = (x, c)$. The norm on $\mathbb{E}$ we consider is $\|\mathcal{X}\|_{\mathbb{E}}^2 = \|x\|_n^2 + \|c\|_{\omega_m^{1,2}}^2$.

2.1 Estimation problem and definitions

Consider the following linear time-varying system driven by an unknown periodic input signal $w(t)$:

$$\dot{x} = A(t)x + A_0(t)w(t), \quad y = C(t)x,$$

where the state $x(t)$ and the output $y(t)$ belong to $\mathbb{R}^n$ and $A(t)$, $A_0(t)$, $C(t)$ are continuous matrices in $\mathcal{M}_{n,n}(\mathbb{R})$, $\mathcal{M}_{n,m}(\mathbb{R})$ and $\mathcal{M}_{n,n}(\mathbb{R})$, respectively, with entries that are uniformly bounded (not necessary periodic), locally integrable functions of t. The matrix $A_0(t)$ has T_0-periodic coefficients. We assume that T_0 is perfectly known, and we want to estimate the T_0-periodic KC^1 (continuous and with piecewise continuous derivative) input signal $t \mapsto w(t) \in \mathbb{R}^m$, with $m = \dim(w) \leq n = \dim(y) = \dim(x)$, through its Fourier decomposition [2] :

$$w(t) \triangleq \sum_{k\in\mathbb{Z}} c_k e^{\imath k\omega_0 t}, \quad \omega_0 = \frac{2\pi}{T_0}$$

In the last expression, each vector c_k admits m complex entries. The state of this model is $\mathcal{X} = (x, c) \in \mathbb{E}$. Because w is real-valued, for any $k \in \mathbb{Z}$, $c_{-k} = c_k^\dagger$. Because w is KC^1, $c \triangleq \{c_k\}_{k\in\mathbb{Z}}$ belongs to $\omega_m^{1,2}$ (as implied by Parseval equality, $\|c\|_{\ell_m^2}^2 = \frac{1}{T_0}\|w\|_{L_m^2[0,T_0]}^2$, and $\|c\|_{\omega_m^{1,2}}^2 = \frac{1}{T_0}\left((1-\frac{1}{w_0^2})\|w\|_{L_m^2[0,T_0]}^2 + \frac{1}{w_0^2}\|w\|_{W_m^{1,2}[0,T_0]}^2\right)$. Simple rewriting yields:

$$\begin{cases} \dot{x} = A(t)x + A_0(t)(\sum_{k\in\mathbb{Z}} c_k e^{\imath k\omega_0 t}) \\ \dot{c}_k = 0, \quad \forall k \in \mathbb{Z} \end{cases}, \quad y = C(t)x \quad (1)$$

Furthermore, following Chauvin and Petit (2010), we make some general assumptions.

H 1. We assume that there exist two strictly positive numbers (ρ_m, ρ_M) such that, for all $t \geq 0$:

$$\begin{cases} A^T(t)A(t) \leq \rho_M^2 I_n \\ \rho_m^2 I_m \leq A_0^T(t)A_0(t) \leq \rho_M^2 I_m \\ \rho_m^2 I_n \leq C(t)C^T(t) \leq \rho_M^2 I_n \end{cases} \quad (2)$$

In particular, we can deduce from $H1$ that $A_0(t)$ has a non-singular pseudo-inverse [3].

2.2 Observer definition

Corresponding to state-space model (1), we define a time-varying Luenberger type observer:

$$\begin{cases} \dot{\hat{x}} = A(t)\hat{x} + A_0(t)(\sum_{k\in\mathbb{Z}} \hat{c}_k e^{\imath k\omega_0 t}) - L(t)(C(t)\hat{x} - y) \\ \dot{\hat{c}}_k = -e^{-\imath k\omega_0 t} L_k(t)(C(t)\hat{x} - y(t)), \quad \forall k \in \mathbb{Z} \\ \left(\hat{x}(0), \hat{c}(0) \triangleq \{\hat{c}_k(0)\}_{k\in\mathbb{Z}}\right) \in \mathbb{E}. \end{cases} \quad (3)$$

The state is $\hat{\mathcal{X}} \triangleq (\hat{x}, \hat{c}) \in \mathbb{E}$. The gain matrices $L(t)$ (with real entries) and $\{L_k\}_{k\in\mathbb{Z}}$ (with complex entries) are defined in the following sub-section [see (4) and (5)].

2.3 Design of L and $\{L_k\}_{k\in\mathbb{Z}}$

By assumption (2), for all $t \geq 0$, $C(t)$ is a square invertible matrix. Let H be a Hurwitz matrix in $\mathcal{M}_{n,n}(\mathbb{R})$. We set

$$L(t) \triangleq (A(t) - H)C^{-1}(t). \quad (4)$$

$\mathbb{R}^+ \ni t \mapsto L(t) \in \mathcal{M}_{n,n}(\mathbb{R})$ is a bounded function because $t \mapsto A(t)$; H and $t \mapsto C^{-1}(t)$ are also bounded. Consider

[1] These inner products implicitly define the norms $\|x\|_{\ell_n^2} =< x, x >_{\ell_n^2}$, and $\|x\|_{\omega_n^{1,2}} =< x, x >_{\omega_n^{1,2}}$.

[2] This is how we account for the periodic nature of the signal w. By making this decomposition early in the study, we are immediately left with an infinite number of variables.

[3] The interested reader might notice that this assumption would enable a least-square approach to estimate $w(t)$ from past measurements of x (from the inversibility of C) through the differential equation (1). Such methods, which usually do not explicitly take advantage of the periodic nature of the signal $w(t)$, are out of the scope of the paper but would be totally relevant here, especially if they are appropriately tuned to account for noises.

P the unique symmetric definite solution in $\mathcal{M}_{n,n}(\mathbb{R})$ of the Lyapunov equation $PH + H^T P = -I_n$. We use P to compute the observer gains as follows: for all $k \in \mathbb{Z}$:

$$L_k(t) \triangleq \frac{\alpha}{k^2+1} A_0^T(t) P C(t)^{-1}, \tag{5}$$

where α is a strictly positive constant. Let $\tilde{\mathcal{X}} \triangleq \mathcal{X} - \hat{\mathcal{X}} \triangleq (\tilde{x}, \tilde{c}) \in \mathbb{E}$. The error dynamics is:

$$\begin{cases} \dot{\tilde{x}} = H\tilde{x} + A_0(t)(\sum_{k\in\mathbb{Z}} \tilde{c}_k e^{ik\omega_0 t}) \\ \dot{\tilde{c}}_k = -\dfrac{\alpha}{k^2+1} e^{-ik\omega_0 t} A_0^T(t) P\tilde{x}, \quad \forall k \in \mathbb{N}, \\ \text{where} \quad (\tilde{x}(0), \tilde{c}(0)) \in \mathbb{E}. \end{cases} \tag{6}$$

Under this form, the roles of the tuning parameters (the Hurwitz matrix H, and the strictly positive constant α) are distinct. H controls the convergence rate of the error state $\tilde{x}$, and α impacts on the convergence of the Fourier coefficient estimates.

From Chauvin and Petit (2010), we have the following convergence result

Proposition 1. Consider system (1). Assume that (2) holds. Consider the observer (3) with L and $\{L_k\}_{k\in\mathbb{Z}}$ defined in (4) and in (5). Then the error dynamics (6) asymptotically converges to zero in $\mathbb{R}^n \times \ell^2_m$.

The frequent case for which A, A_0 and C are time-invariant is addressed in the following corollary.

Corollary 1. Consider system (1). Assume that C is invertible and A_0 is injective. Consider the observer (3) with L and $\{L_k\}_{k\in\mathbb{Z}}$ defined in (4) and in (5). Then the error dynamics (6) asymptotically converges toward $0_{\mathbb{R}^n\times\ell^2_m}$.

3. SENSOR DYNAMIC INVERSION

A first example is the inversion of sensor dynamics (see Hammerschmidt and Leteinturier (2004) and Heywood (1988) for more details). A classic model of such a sensor is a first order dynamics with periodic excitation which can be, depending on the application, the intake pressure, the intake temperature, the exhaust pressure, the air fuel ratio, or the mass air flow ...

For tutorial purpose, the system under consideration is

$$\begin{cases} \dot{x} = \dfrac{1}{\tau}(w(t) - x) \\ w(t) = \sum_{k\in\mathbb{Z}} c_k e^{\imath kt} \end{cases}$$

with the coefficients $\{c_k\}_{k\in\mathbb{Z}}$ defined by

$$c_k = \begin{cases} \dfrac{1+i}{2(1+k^2)} & \text{for} \quad k > 0 \\ 1 & \text{for} \quad k = 0 \\ \dfrac{1-i}{2(1+k^2)} & \text{for} \quad k < 0 \end{cases}$$

Observer design The main interest of this simple example is to show that even if the number of harmonics under consideration is high, the observer design remains easy and is robust with respect to noise. The observer we propose is, following (3),

$$\begin{cases} \dot{\hat{x}} = \dfrac{1}{\tau}(\sum_{k\in\mathbb{Z}} \hat{c}_k e^{\imath kt} - \hat{x}) - L_x(x - \hat{x}) \\ \dot{\hat{c}}_k = \dfrac{\alpha}{2(k^2+1)} e^{-\imath kt}(\hat{x} - x), \quad \forall k \in \mathbb{Z} \\ \hat{x}(0) = 0, \hat{c}_k(0) = 0, \ \forall k \in \mathbb{Z}. \end{cases}$$

In this example, there are only two tuning parameters for this infinite dimensional problem. The roles of the tuning parameters (the gain L_x, and the strictly positive constant α) are distinct. L_x controls the convergence rate of the error state $\hat{x}$, and α impacts on the convergence of the Fourier coefficient estimates.

Simulation results In practice, only a finite number of Fourier expansion coefficients can be included. However, numerous harmonics often need to be considered to reconstruct the signal. In the numerical application, we use $\tau = 4$, $L_x = 1$ and $\alpha = 0.5$. To show the relevance of the approach, we use 100 harmonics [4]. In Figure 1, we show the square error between $\hat{w}$ and w with respect to time.We can see that convergence is provided even with a large number of variables. Moreover, we see the decreasingness of the L^2-norm of the Fourier decomposition as time goes to infinity.

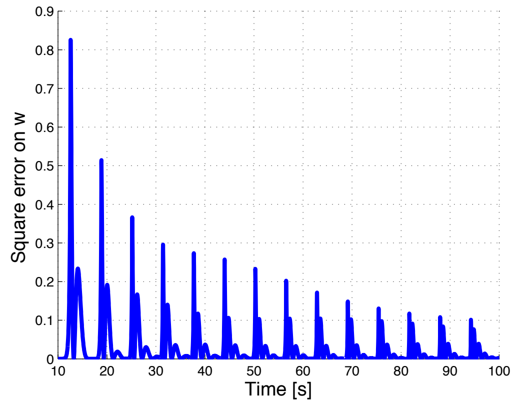

Figure 1. $\|\hat{w} - w\|^2$ on a test with 100 harmonics.

Finally, the observer design is very robust to measurement noise. In Figures 2 and 3, we present results with a measurement gaussian noise with a standard deviation of 0.4. Robustness is guaranteed while preserving convergence of the state, the periodic input and its Fourier decomposition.

4. CRANKSHAFT DYNAMICS REFERENCE MODEL

We now present a second automotive system observation problem, also considered in Rizzoni (1989); Chauvin et al. (2004); van Nieuwstadt and Kolmanovsky (1997). Consider an n_{cyl}-cylinder engine. Following Kiencke and Nielsen (2000), the torque balance on the crankshaft can be written as $\frac{d\left(\frac{1}{2}J(\theta)\omega^2\right)}{d\theta} = T$, where θ is the crank angle, ω is the instantaneous engine speed, J is the $\frac{4\pi}{n_{cyl}}$-periodic inertia, and T is the combustion torque. In the variable θ time scale, T is $\frac{4\pi}{n_{cyl}}$-periodic and has zero mean at steady state. This system defines a first-order periodic dynamics with a periodic input signal T. The state $x(\theta) = \frac{1}{2}J(\theta)\omega^2$

[4] It means that we restrict k to [-100, 100]

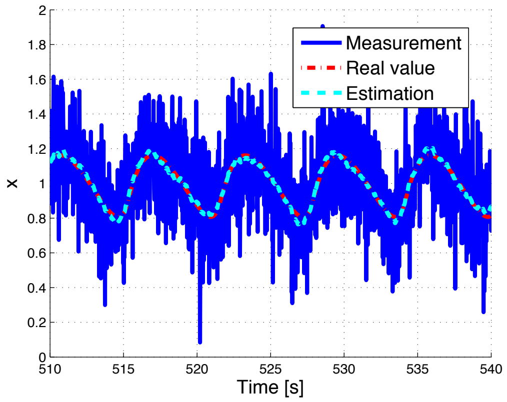

Figure 2. Test with 100 harmonics. Measurement, real value of x and estimated value of x.

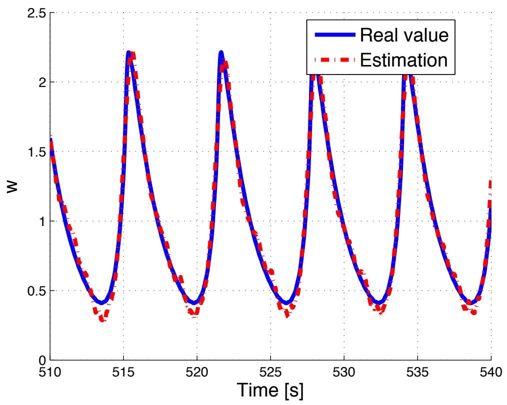

Figure 3. Test with 100 harmonics. w and its estimation ($\hat{w}$).

is fully measured through the equation $y = \omega^2$. The periodic input signal T admits a Fourier series expansion $T \triangleq \sum_{k\in\mathbb{Z},k\neq 0} c_k e^{\imath k\frac{\theta}{2}}$. The reference dynamics is

$$\frac{dx}{d\theta} = \sum_{k\in\mathbb{Z},k\neq 0} c_k e^{\imath k\frac{\theta}{2}}$$

Then, the observer we propose is, following (3),

$$\left\{\begin{aligned} \frac{d}{d\theta}\hat{x} &= \sum_{k\in\mathbb{Z},k\neq 0} \hat{c}_k e^{\imath k\frac{\theta}{2}} + H(\hat{x} - \frac{1}{2}J(\theta)y) \\ \frac{d}{d\theta}\hat{c}_k &= \frac{\alpha}{2(k^2+1)H} e^{-\imath k\frac{\theta}{2}}(\hat{x} - \frac{1}{2}J(\theta)y), \quad \forall k \neq 0 \end{aligned}\right.$$
$$\hat{x}(0) = 0, \hat{c}_k(0) = 0, \ \forall k \neq 0. \tag{7}$$

Assumption (2) is easily verified with

$$\left\{\begin{aligned} \rho_m &= \min\{1, \min_{\theta\in[0,\frac{4\pi}{n_{cyl}}]} \frac{2}{J(\theta)}\} \\ \rho_M &= \max\{1, \max_{\theta\in[0,\frac{4\pi}{n_{cyl}}]} \frac{2}{J(\theta)}\} \end{aligned}\right.$$

To estimate T, we can use the observer (7) with, e.g., $H = -100$ and $\alpha = 50$ (these are the values used to obtain the experimental results presented below). This gives the estimate $\hat{T} = \sum_{k\in\mathbb{Z},k\neq 0} \hat{c}_k e^{ik\frac{\theta}{2}}$.

Experimental results In practice, only a finite number of Fourier expansion coefficients can be included. However, numerous harmonics need to be considered to reconstruct the signal (at least 5). Very conveniently, the observer design can easily be updated when the number of harmonics considered is changed. Indeed, without modifying the tuning parameters H and α, new gains are computed from (4) and (5). These formulae remain valid when the number of harmonics asymptotically approaches infinity. Figure 4 shows experimental observer results for a four-cylinder diesel engine. It is possible to compare our results to a high-accuracy estimate obtained from in-cylinder pressure sensors.

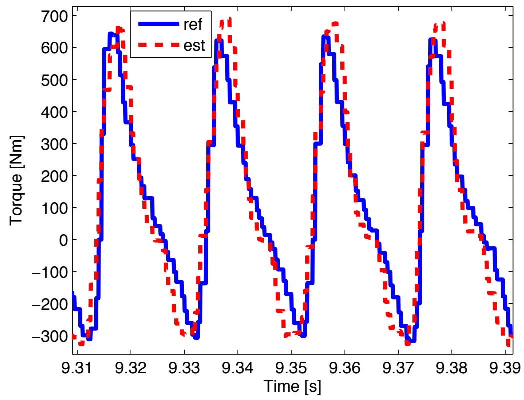

Figure 4. Combustion torque. Continuous line, reference combustion torque obtained from in-cylinder pressure sensors; dashed line, combustion torque estimated by the proposed observer.

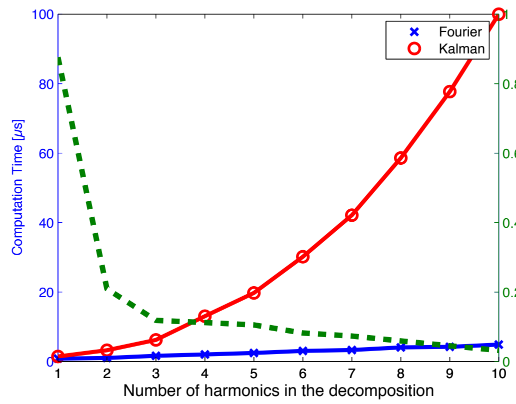

Figure 5. Normalized CPU time as a function of the number of terms considered in the Fourier expansion. Comparison between an extended Kalman filter and the proposed observer (3). Right axis, reconstruction error.

The results for the observer presented are good. For similar accuracy, the computational burden of our observer compares very favorably to an extended Kalman filter (as previously used Chauvin et al. (2004)). The results in Figure 5 demonstrate that this burden increases quadratically as a function of the number of harmonics for the extended Kalman filter, whereas it simply increases linearly for our observer. The interested reader will also note that another advantage of the proposed method is its proof of convergence. Interestingly, convergence of the EKF for this system can also be established (see Chauvin et al. (2004)), but it requires a careful investigation of observability and controllability Grammians to guarantee uniform (with respect to the time variable) properties of this periodic system. These properties guarantee existence and uniqueness of a symmetric periodic positive solution to the discrete periodic Riccati equation that serves in the proof of convergence.

Besides its convergence, the tuning simplicity and the relatively low computational cost are the two points of interest of the proposed technique.

5. OSCILLATING WATER COLUMN

We now wish to study a third example. Among all the oceans renewable energy resources, wave energy is one of the most promising and consequently one of the most studied currently. This resource have been evaluated, and it appears that, while the annual average power density (aapd) is very high at certain locations (i.e. aapd$> 20kW$ for 1 m of wave front), the recovery of this energy is made a difficult challenge by the large dispersion of energy over the energy spectrum. An overview of the state of art in recovering ocean wave energy can be found in Brook (2003). The oscillating water column (OWC) power plant represents the most studied device today; many prototypes and projects have been built on this principle all over the world [5].

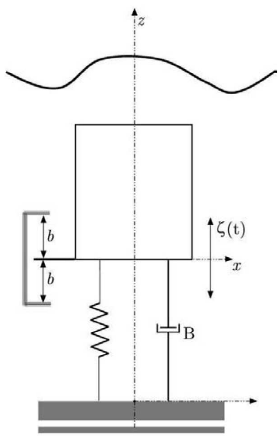

Figure 6. Generic device of a oscillating water column.

In this study, we shall focus on the control of a generic point absorber device with a single degree of freedom (DoF). A generic device is presented in Figure 6. Namely, we will consider a submerged vertical cylinder constrained to move in heave motion only, under the action of wave forces. Yet, all theoretical work presented here can be applied to the more common case of floating bodies (provided the linearized buoyancy force is included in the spring force). The body oscillates vertically under the action of : excitation forces, radiation forces, restoring forces idealized here as a single spring of stiffness k, and a damping force proportional to the velocity (with damping coefficient B) supposed to represent the action of the external Power Take Off (PTO) mechanism. The vertical motion around the equilibrium position will be denoted by $\zeta(t)$. A linear approach will be adopted here for modelling the hydrodynamics (see Josse and Clément (2007); Porter and Evans (1995)), in such a way that the behavior of the body in waves is governed by the following differential equation [6]

$$(M+\mu_\infty)\ddot{\zeta}+B\dot{\zeta}+K\zeta(t)=F_{ex}(t)+u$$

where F_{ex} is the external force and u the control input. The waves can be considered (as a first approximation) as a periodic motion with a known frequency (see Pitt and Tucker (2001); Laitone and Wehausen (1960) for example). The forecast this wave is of high importance. Indeed, for control purposes (see Chatry et al. (1998) for example), one would try to come in resonance with the wave to maximise the recuperated power.

System model (for estimation purpose) For clarity, the system is rewritten under the form (1) with $x=[\zeta,\ \dot{\zeta}]$ and

$$A=\begin{bmatrix}0 & 1\\ -2\xi\omega_{ref} & -\omega_{ref}^2\end{bmatrix},\ A_0=\begin{bmatrix}0\\1\end{bmatrix}$$
$$C=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

We define the wave by coefficients scaled by a factor of 10 and $c_0=0$ (zero mean wave). The $\{c_k\}_{k\in\mathbb{Z}}$ are defined by

$$c_k=\begin{cases}\dfrac{1+i}{20(1+k^2)} & \text{for} \quad k>0\\ 0 & \text{for} \quad k=0\\ \dfrac{1-i}{20(1+k^2)} & \text{for} \quad k<0\end{cases}$$

Simulation results In this example, we used $\xi=.1$ and $\omega_{ref}=1.25$. The proposed observer is defined by (3).

There are only two tuning parameters for the infinite dimensional problem. The roles of the tuning parameters (the gain H, and the strictly positive constant α) are distinct. H controls the convergence rate of the error state $\hat{x}$. We used $H=diag(1,1)$ in the simulation result. The parameter α impacts on the convergence of the Fourier coefficient estimates (calibrated at 0.1 in the following results).

In practice, only a finite number of Fourier expansion coefficients can be included. However, numerous harmonics often need to be considered to reconstruct the signal. To show the relevance of the approach, we use 100 harmonics.

The observer design is very robust toward measurement noise (both on ζ and $\dot{\zeta}$). In Figures 7 and 8, we present results with a measurement gaussian noise with a standard deviation of 0.04. Robustness is guaranteed while preserving convergence of the state, the periodic input and its Fourier decomposition. One can see that convergence is provided even with a large number of variables. Moreover,

[5] One can cite the Pelamis (www.pelamiswave.com), the AWS (www.awsocean.com), and the SEAREV (fr.wikipedia.org/wiki/Searev)

[6] A more precise model leads to a more complex damping coming from the Cummins decomposition (see Cummins (1962)) of the radiation forces. For sake of simplicity, this has not been added in the paper.

the L^2-norm of the Fourier decomposition decreased over time (as can be seen in Figure 9).

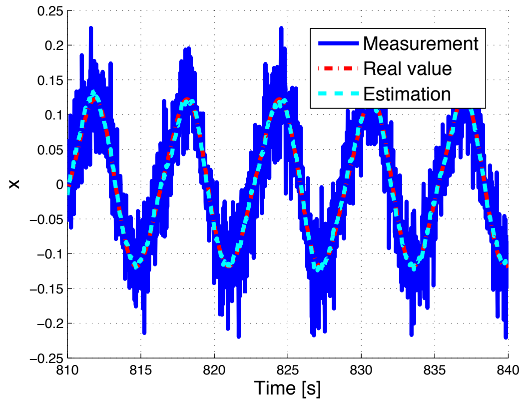

Figure 7. Test with 100 harmonics. Measurement, real value of ζ, and its estimation $\hat{\zeta}$.

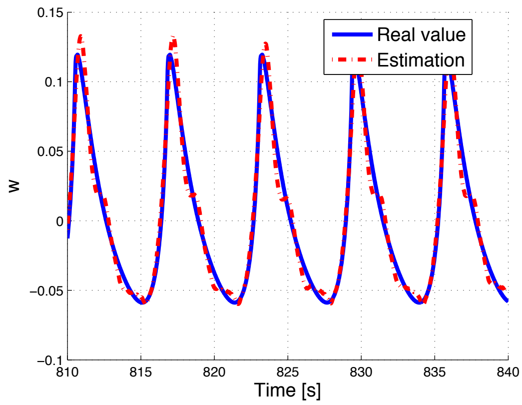

Figure 8. Test with 100 harmonics. w and its estimate $(\hat{w})$.

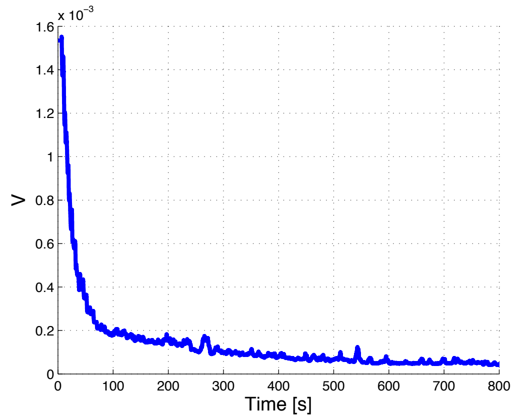

Figure 9. $V = \sum_{k \in [-100,100]} \|\hat{c}_k - c_k\|^2$ on a test with 100 harmonics.

6. CONCLUSION

Several examples of engineering interest have been presented in this paper to show the merits of a recently proposed technique to asymptotically reconstruct the Fourier expansion of periodic input signals of linear time varying systems. The main characteristic are its ease of tuning, even when large number of harmonics coefficients are considered, its low computational complexity, and its convergence. As appears in the example treated, it is relatively robust to noise in the measurements which is an appealing feature for real applications.

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