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# Finite Element Modeling of Superplastic Sheet Forming Processes. Identification of Rheological and Tribological Parameters by Inverse Method

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**Abstract.** Superplastic forming is a thermoforming-like process commonly applied to titanium and aluminum alloys at high temperature and in specific conditions. This paper presents the application of an inverse analysis technique to the identification of rheological and tribological parameters. The method consists of two steps. First, two different kinds of forming tests have been carried out for rheological and tribological identification, using specific mold shapes. Accurate instrumentation and measurements have been done in order to feed an experimental database (values of appropriate observables). In a second step, the development of an inverse method has been carried out. It consists of the minimization of an objective function representative of the distance - in a least squares sense - between measured and calculated values of the observables. The algorithm, which is coupled with the finite element model FORGE2®, is based on a Gauss-Newton method, including a sensitivity matrix calculated by the semi-analytical method.

## INTRODUCTION

Superplastic forming (SPF) is a production process for integrated metallic structures. The most common alloys being used are aluminum and titanium alloys. The process consists in applying a differential pressure between sides of a heated sheet to shape it in a mold, according to a thermoforming technique. One of the key problems during processing is the fulfillment of a prescribed thickness distribution of the final part. Obviously numerical simulation can help in process optimization [1]. Finite element codes, like TFORM3® at Cemef [2], have been developed in that sense. It is based on a thin shell approach in three-dimensional space, which is well suited for the blowing of single thin sheets. In this study, the code FORGE2® has been used. It offers a two-dimensional full volumic description which is appropriate for an accurate analysis of thin sheet forming, or for thick sheet forming, or for the more complex SPF/DB process (involving several sheets partially welded by a diffusion bonding technique prior to processing).

Despite the progress of numerical models, the accuracy of SPF-simulations depends on the relevance of input physical parameters, especially rheological and tribological ones. Regarding superplastic behavior, it is generally investigated through tensile tests although it is known that the response obtained in pure tensile conditions is not representative enough of the effective mechanical solicitation in SPF process, which is biaxial stretching. This has motivated the use of the biaxial bulging test as a reference rheological test for SPF [3]. About friction behavior, no specific studies have been carried out so far, that would be representative enough of the real processing conditions. The development of representative rheological and tribological tests in turn induces another problem: such tests simply cannot be analyzed by analytical models. In order to avoid painful and inefficient trial-and-error procedures, automatic inverse analysis techniques have been developed for the identification of constitutive parameters [4-5]. In this study, this work is continued and extended to the identification of friction parameters.

# GOVERNING EQUATIONS

## Constitutive Equations

Fine grain size and high temperature (compared to the melting temperature) are the basic requirements for superplasticity [6-7]. The main characteristic of the superplastic behavior is the high sensitivity of the flow stress to the strain rate, which ensures deformation stability. This phenomenon occurs in a certain range of low strain rates. The flow stress has then low values and the flow rule is of the following form:

$$\sigma_{eq} = k(\varepsilon_{eq})\dot{\varepsilon}_{eq}{}^{m(\varepsilon_{eq})} \tag{1}$$

where $\sigma_{eq}$ is the equivalent von Mises' stress, $\dot{\varepsilon}_{eq}$ the equivalent strain rate, $\varepsilon_{eq}$ the equivalent strain, $k$ the consistency, $m$ the strain rate sensitivity index.

The dependence of $k$ and $m$ with respect to $\varepsilon_{eq}$ involves several parameters, the set of which will be denoted $\boldsymbol{q}$:

$$\boldsymbol{q} = \left[q_1, q_2, \cdots, q_p\right]^{\mathrm{T}} \tag{2}$$

## Boundary Conditions

The domain occupied by the material at a given instant of the process is denoted $\Omega$ and its boundary $\partial\Omega$.

*Pressure load:* when a region of $\partial\Omega$ is submitted to an applied pressure $P$, the local stress vector is:

$$\boldsymbol{T} = \boldsymbol{\sigma n} = -P\boldsymbol{n} \tag{3}$$

where $\boldsymbol{\sigma}$ is the stress tensor and $\boldsymbol{n}$ the outward normal vector.

*Contact:* SPF tools can be considered perfectly rigid. The usual equations for unilateral contact must be satisfied:

$$\begin{cases} \boldsymbol{\sigma n} \cdot \boldsymbol{n} \le 0 \\ (\boldsymbol{v} - \boldsymbol{v}_{tool}) \cdot \boldsymbol{n} \le 0 \\ ((\boldsymbol{v} - \boldsymbol{v}_{tool}) \cdot \boldsymbol{n})(\boldsymbol{\sigma n} \cdot \boldsymbol{n}) = 0 \end{cases} \tag{4}$$

In case of contact, friction should be taken into account.

## Friction Models

In superplastic forming a lubricant is used in order to minimize friction, but also to avoid adhesion and make the removal from the mold easier. The most common lubricant used is boron nitride, which is also used as a diffusion barrier (called "stop-off") in the SPF/DB process. The applied stresses being low, different friction models of Coulomb type can be considered.

The standard Coulomb law relates the tangential stress vector to the normal stress:

$$\boldsymbol{T}_\tau = \mu_f \sigma_n \frac{1}{\left\| \boldsymbol{v} - \boldsymbol{v}_{tool} \right\|}(\boldsymbol{v} - \boldsymbol{v}_{tool}) \tag{5}$$

In this work, we have studied an extended model proposed by Kato and Hirasawa [8], in which the friction coefficient $\mu_f$ depends on the strain rate:

$$\boldsymbol{T}_\tau = (\mu_0 + c \ln \dot{\varepsilon}_{eq})\sigma_n \frac{1}{\left\| \boldsymbol{v} - \boldsymbol{v}_{tool} \right\|}(\boldsymbol{v} - \boldsymbol{v}_{tool}) \tag{6}$$

## Mechanical Equilibrium

The local momentum equation to be solved is:

$$\nabla \cdot \boldsymbol{\sigma} + \rho \boldsymbol{g} = \rho \boldsymbol{\gamma} \tag{7}$$

where $\rho$ denotes the density, $\boldsymbol{g}$ the gravity vector and $\boldsymbol{\gamma}$ the acceleration vector. The equivalent weak form is given by the principle of virtual power:

$$\forall \boldsymbol{v}^* \quad \int_\Omega \boldsymbol{\sigma} : \nabla \boldsymbol{v}^* dV - \int_\Omega \rho \boldsymbol{g} \cdot \boldsymbol{v}^* dV + \int_{\partial\Omega_p} P\boldsymbol{n} \cdot \boldsymbol{v}^* dS - \int_{\partial\Omega_c} \boldsymbol{T}_\tau \cdot \boldsymbol{v}^* dS + \int_\Omega \rho \boldsymbol{\gamma} \cdot \boldsymbol{v}^* dV = 0 \tag{8}$$

where $\partial\Omega_p$ denotes the part of the boundary where pressure applies and $\partial\Omega_c$ the contact region. This equilibrium condition must be fulfilled under incompressibility condition $\nabla \cdot \boldsymbol{v} = 0$, whose weak form is:

$$\forall p^* \quad \int_\Omega p^* \nabla \cdot \boldsymbol{v} \, dV = 0 \tag{9}$$

Regarding the treatment of contact boundary conditions, in addition to the above mentioned expression of the tangential stress vector $\boldsymbol{T}_\tau$, the nonpenetration condition (4) is enforced by a penalty approach, in which the following contribution is added to the left hand side of (8):

$$\int_{\partial\Omega_c} \chi_p [(\boldsymbol{v} - \boldsymbol{v}_{tool}) \cdot \boldsymbol{n}]^{+} \boldsymbol{n} \cdot \boldsymbol{v}^{*} dS \qquad (10)$$

The constant $\chi_p$ is arbitrary large and the bracket $[x]^+$ denotes the positive part of $x$: $[x]^+=x$ if $x>0$ and $[x]^+=0$ if $x<0$.

After time discretization of inertia terms, spatial discretization of equations (8) and (9), and resolution for $\boldsymbol{v}$ and $p$, the configuration $\Omega$ is updated as follows:

$$\boldsymbol{x}^{t+\Delta t} = \boldsymbol{x}^{t+\Delta t} + \Delta t \boldsymbol{v} + \frac{1}{2}\Delta t^2 \boldsymbol{\gamma} \qquad (11)$$

## INVERSE MODEL

The principle of the method is to adjust the values of the parameters in order to minimize the difference between numerical calculations and measurements. For this purpose, a series of experiments have to be selected, that should be representative of the process conditions. For each type of experiment, some observables $H$ have to be carefully defined on the basis of their sensitivity to the material parameters to be identified. Denoting $H^{exp}$ the measured values and $H^{cal}$ the calculated ones, a sampling procedure has to be defined in order that the vectors $\boldsymbol{H}^{exp}$ and $\boldsymbol{H}^{cal}$ of $H$ values on the whole series of experiments can be compared. At this stage, an essential task is to get precise and reproducible measurements. The calculated values must be deduced from the time incremental resolution of mechanical equations (8) and (9), which is called the direct model. Those values $H^{cal}$ of course depend on the input material parameters $\boldsymbol{q}$.

The inverse optimization problem consists then of the determination of the set $\boldsymbol{q}$ minimizing the distance between the sets of experimental $\boldsymbol{H}^{exp}$ and calculated values $\boldsymbol{H}^{cal}$. This distance is expressed in the sense of the classical least squares method:

$$\Psi(\boldsymbol{H}^{cal}(\boldsymbol{q}), \boldsymbol{H}^{exp}) = \sum_{l=1}^{s} \beta_l (H_l^{\,cal}(\boldsymbol{q}) - H_l^{\,exp})^2 \qquad (12)$$

where $s$ is the number of total sampled points and $\beta_l$ is the weight function associated to point $l$. A Gauss-Newton method is used to solve the problem. At iteration $\nu$, the new estimate of the set of parameters $\boldsymbol{q}$ is given by:

$$\boldsymbol{q}^{(\nu)} = \boldsymbol{q}^{(\nu-1)} - \left[\boldsymbol{J}(\boldsymbol{q}^{(\nu-1)})\right]^{-1} \frac{\partial \Psi}{\partial \boldsymbol{q}}(\boldsymbol{q}^{(\nu-1)}) \qquad (13)$$

$$\text{with} \quad \begin{cases} \dfrac{\partial \Psi}{\partial q_i} = 2\displaystyle\sum_{l=1}^{s} \beta_l \left(H_l^{cal}(\boldsymbol{q}) - H_l^{exp}\right) \dfrac{\partial H_l^{cal}}{\partial q_i} \\ J_{ij} \cong 2\displaystyle\sum_{l=1}^{s} \beta_l \dfrac{\partial H_l^{cal}}{\partial q_i} \dfrac{\partial H_l^{cal}}{\partial q_j} \end{cases} \qquad (14)$$

The Jacobian matrix $\boldsymbol{J}$ depends then on the sensitivity matrix $\boldsymbol{S} = \partial \boldsymbol{H}^{cal} / \partial \boldsymbol{q}$. This matrix can be evaluated either by finite difference method [9], analytical differentiation [5], or by semi-analytical differentiation [10]. The latter method has been used in this study because it offers at the same time an easy implementation and a sound, fast and robust optimization behavior. It should be noted that $\boldsymbol{J}$ is an approximation of the Jacobian matrix, neglecting second order derivatives. Therefore the iterative scheme (13) is complemented with a line-search procedure in order to increase its robustness [11].

## EXPERIMENTAL SET-UP

Figure 1 shows the forming press used at ENSAM Angers. It can form by argon pressure aluminum, titanium and nickel alloys. The maximum pressure is 12 MPa and the maximum temperature 1300 °C. After initial heating in the machine and clamping at their periphery, circular blanks are formed in molds made of refractory steels (maximum mold diameter ≈ 300 mm). An inductive position sensor (precision 0.1 mm) associated with a ceramic rod permits the measurements of the dome apex, in case of axisymmetric bulging.

**FIGURE 1.** Experimental setup at ENSAM Angers.

# IDENTIFICATION OF CONSTITUTIVE PARAMETERS

## Conical Forming Tests

A cone-shaped mold has been selected, as shown in Fig. 2. Such a geometry is well suited for rheological identification as the influence of friction condition is very low. This has been investigated by numerical simulation [12]: the advancement of the forming is quasi identical whatever the friction parameters. The selected observable $H$ in this case is then the time evolution of the cone height, as measured by the sensor.

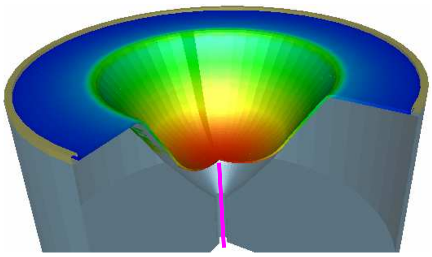

**FIGURE 2.** Conical geometry of the test with the deformed sheet and its strain distribution, as calculated by FORGE2® in axisymmetric 2D conditions. In reality, the sheet is formed upwards and the height sensor is pushed upwards.

## Titanium Alloy

The method has been applied to the identification of the constitutive parameters $k$, $m$ and $n$ for the following law:

$$\sigma_{eq} = k\varepsilon_{eq}^{\ n}\dot{\varepsilon}_{eq}^{\ m} \qquad (15)$$

Five conical forming tests using 2 mm thick titanium alloy sheets (Ti-6Al-4V alloy) have been carried out, one of them including pressure variations in time, resulting in variable bulging speed (Fig. 3). Using this experimental database, the identification failed for non convergence in the Gauss-Newton procedure. The reason is a too strong correlation between parameters $k$ and $m$, as analyzed in [13, 11]. If the parameter $m$ is fixed to a given value, then the two other parameters $k$ and $n$ can be identified. As shown in Table 1, the iterative procedure leads to a stagnation of the objective function after 4 iterations only. The 4.60 % error ratio obtained is illustrated in Fig. 3, which shows the difference between measured and calculated cone height evolutions.

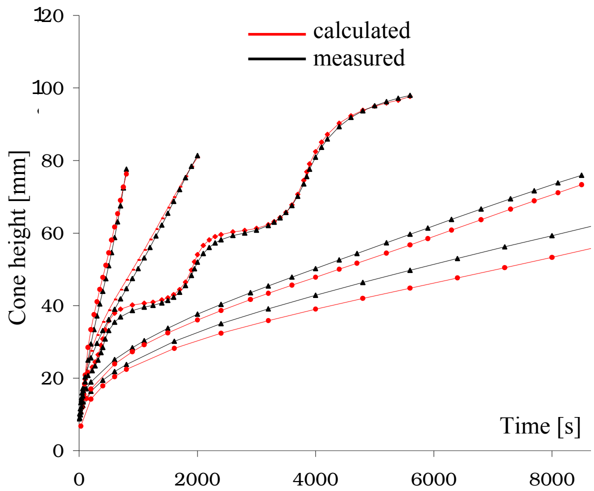


**FIGURE 3**. Identification of Ti-alloy constitutive parameters with 5 conical forming tests, during which the cone height evolution has been measured.

**TABLE 1. Ti-alloy, identification of constitutive parameters (*m* fixed): evolution of the objective function.**

| Iterations | Objective function (%) |
|---|---|
| 0 | 10.26 |
| 1 | 7.58 |
| 2 | 4.96 |
| 3 | 4.63 |
| ≥ 4 | 4.60 |

## Aluminum Alloy

The method has been applied to the identification of the parameters of the constitutive law (15) for 4-mm thick aluminum alloy sheets (7475 alloy) deformed at 510 °C. Four conical forming tests have been done, each of them including pressure variations in time, resulting in variable bulging speed, as illustrated in Fig. 4. In this case, the same trend has been found: strong $k$-$m$ correlation; convergence reached only when $m$ fixed. We have then enriched the experimental database with two results obtained by direct simulations of tests with a different pressure evolution, using the values of $k$ and $n$ identified previously and the fixed value of $m$. Using this augmented database, the concurrent identification of the three parameters $k$, $n$ and $m$ has been successful. The convergence, shown in Table 2, is good, leading to a stagnation after 5 iterations. Figure 5 shows the good agreement obtained finally. This shows that when two parameters are strongly correlated, an extension of the experimental database is necessary to get a sound convergence of the identification procedure.

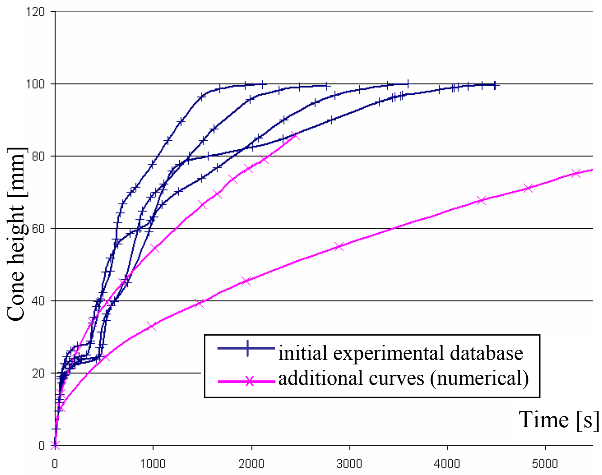


**FIGURE 4**. Identification of Al-alloy constitutive parameters. The experimental database (4 conical forming tests, in blue color) has been complemented by two numerical direct simulations (in purple), in order to permit concurrent identification of the three parameters *k*, *n* and *m*.

**TABLE 2. Al-alloy, identification of constitutive parameters: evolution of the objective function.**

| Iterations | Objective function (%) |
|---|---|
| 0 | 14.45 |
| 1 | 10.14 |
| 2 | 9.27 |
| … | … |
| ≥ 5 | 5.21 |

# IDENTIFICATION OF FRICTION PARAMETERS

## Selected Geometry

A specific test has been designed for friction identification in superplastic forming conditions [14]. Like the rheological test, it consists of a real superplastic forming, but its geometry has been especially studied in order to be selective with respect to friction parameters (Fig. 6). During forming, the central insert is progressively wrapped by the sheet. The free regions of the sheet then tend to stretch the central region and make it slide along the insert. The thickness evolution being highly dependent on contact conditions, the selected observable is the thickness profile. The sampling consists of 20 thickness measurements along a radial section of each formed part.

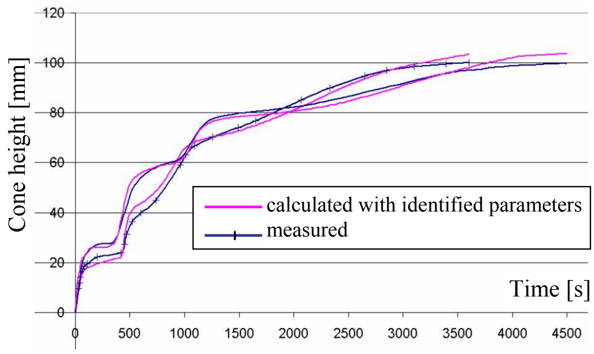


**FIGURE 5**. Identification of Al-alloy constitutive parameters. Good agreement on cone height evolution after parameter identification.

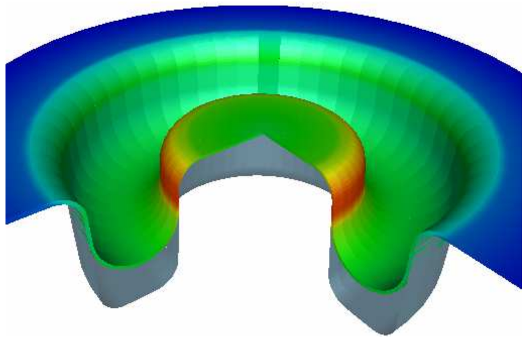

**FIGURE 6.** Specific geometry for friction identification. Deformed sheet and its strain distribution, as calculated by FORGE2® in axisymmetric 2D conditions.

## Identification

The parameters of the strain-rate dependent friction law (6) have been identified, in the case of aluminum alloy. Different forming tests have been done at a temperature of 470 °C and for maximum strain rates in the range [8 $10^{-5}$, 5 $10^{-4}$]. As shown in Table 3, the identification of the two parameters is obtained after 8 iterations, yielding a residual error of about 8 %. Figure 7 gives an example of the difference found between calculated and measured thickness distribution.

**TABLE 3. Al-alloy, identification of friction parameters: evolution of the objective function.**

| Iterations | Objective function (%) |
|---|---|
| 0 | 32.25 |
| 1 | 19.90 |
| 2 | 12.95 |
| … | … |
| ≥ 8 | 8.46 |

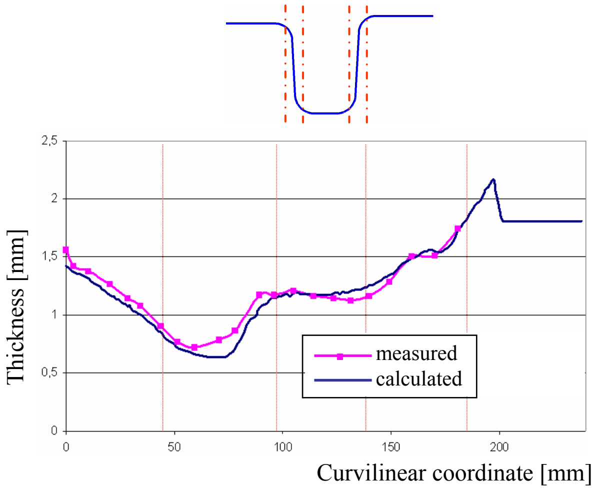


**FIGURE 7.** Example of comparison between calculated and measured thickness profiles after parameter identification. The 4 vertical lines separate the different regions of the sheet, as indicated on the top schematic.

## CONCLUSION

An inverse method has been applied to identify constitutive and friction parameters in superplastic forming conditions. The experimental work has been conducted using two specific axisymmetric mold shapes. The identification procedure has consisted of a Gauss-Newton algorithm using semi-analytic derivatives, coupled with the FEM code FORGE2®. In the future this methodology will be applied to the identification of more complex constitutive and friction equations.

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