ACTIVE CONTROL AND MOTION PLANNING FOR OFFSHORE STRUCTURES

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This paper presents an active control dedicated to the positioning of vertical offshore structures. The trajectory planning and the closed loop system use a convenient model given by the Bernoulli's historical cable equation, completed with a damping factor, that linearly depends on the structure speed. Its solution is directly used in the control design, providing an extension to previous works on control of heavy chains and offshore structures.

keywords: Re-entry operation, Motion Planning, Flexible Structures

Introduction

This paper presents an active control dedicated to a re-entry problem found in the offshore oil industry. The re-entry operation consists in connecting the bottom of a very long pipeline to the wellhead, by dynamically modifying the pipeline top position, which is linked to a floating device (vessel or platform). These long pipelines are usually called risers, because they are used to rise the drilling mud or the hydrocarbons from the wellhead to the platform. Nowadays the re-entry operation is done manually. The use of an active control intends to reduce the operation time, and to make it possible even under bad weather conditions. The considered offshore structure can be analyzed as a cable submerged in a flow. A convenient model is given by the Bernoulli's historical cable equation, completed with a damping factor, that linearly depends on the structure speed. The damping factor is developed in series around zero, to get an approximate solution. The corresponding model turns out to be differentially flat[1], a property directly used in the control design, providing an extension to previous works of Petit and Rouchon [2], Thull et al [3], and Sabri [4]. The presented solution permits to calculate the reference trajectory for the riser top position, from a pre-defined reference trajectory for the riser bottom position. A tracking controller is used to ensure that the trajectories are followed with stability. Numerical simulations with two different disturbances are presented.

1 Governing equations

Generally, offshore structures are slender and their transverse displacements are small when compared to their height, so they can be analyzed as a linearized Euler-Bernoulli beam with a constant section, under an axial traction plus external forces from the fluid [5]:

$$m_s \frac{\partial^2 \Upsilon}{\partial t^2} = -EJ\frac{\partial^4 \Upsilon}{\partial z^4} + \frac{\partial}{\partial z}\left(T\frac{\partial \Upsilon}{\partial z}\right) + F(z,t) \tag{1}$$

Equation (1) represents the dynamic behavior of the structure in the direction of its main horizontal displacement. The variable Υ represents the structure displacement in this direction and is a function of two parameters: the time t and z, representing the height from the seabed. The other variables are the linear mass m_s, the mechanical tension T, the Young's modulus E, the second moment of inertia J, and the hydrodynamic forces in this direction, represented by F. The Morison's equation defines the hydrodynamic forces associated to a relative displacement of a submerged body in a fluid (see [5]):

$$F(z,t) = -m_F\frac{\partial^2 \Upsilon}{\partial t^2} - \mu\frac{\partial \Upsilon}{\partial t}\left|\frac{\partial \Upsilon}{\partial t}\right| \tag{2}$$

Considering μ as the drag constant and m_F as the fluid added mass, denoting $m = m_S + m_F$, we can rewrite equation (1) as:

$$m\frac{\partial^2 \Upsilon}{\partial t^2} = -EJ\frac{\partial^4 \Upsilon}{\partial z^4} + \frac{\partial}{\partial z}\left(T\frac{\partial \Upsilon}{\partial z}\right) - \mu\frac{\partial \Upsilon}{\partial t}\left|\frac{\partial \Upsilon}{\partial t}\right| \tag{3}$$

In the case of the re-entry operation, the displacement has low frequencies, so that $\frac{\partial}{\partial z}\left(T\frac{\partial \Upsilon}{\partial z}\right) \gg -EJ\frac{\partial^4 \Upsilon}{\partial z^4}$ (for low natural modes, the beam effects can be neglected). The tension for a disconnected riser in these conditions is a linear function of its weight and can be defined as $T = (m_s - \rho S)z$, where ρ represents the water density and S the transverse section surface. Dividing equation (3) by m we get:

$$\frac{\partial^2 \Upsilon}{\partial t^2} = +\frac{\partial}{\partial z}\left(\frac{(m_s-\rho S)z}{m}\frac{\partial \Upsilon}{\partial z}\right) - \frac{\mu}{m}\frac{\partial \Upsilon}{\partial t}\left|\frac{\partial \Upsilon}{\partial t}\right|$$

The constant term $(m_s - \rho S)/m$ can be replaced by the effective gravity g. It is proposed to linearize the drag term, substituting the term $\frac{\mu}{m}\left|\frac{\partial \Upsilon}{\partial t}\right|$ by the constant τ, that is calculated as a function of μ/m and of the mean value of $\frac{\partial \Upsilon}{\partial t}$ along the structure. With this approximation the system equation becomes the cable equation defined by Bernoulli (see [2]) plus a linear damping factor:

$$\frac{\partial^2 \Upsilon}{\partial t^2}(z,t) = \frac{\partial}{\partial z}\left(gz\frac{\partial \Upsilon}{\partial z}(z,t)\right) - \tau\frac{\partial \Upsilon}{\partial t}(z,t) \tag{4}$$

2 Motion planning

In this section, we present an analytical solution of equation (4) in the Laplace domain, following the idea proposed by Petit and Rouchon [2]. This solution is developed in Taylor's series, in order to formally inverse the Laplace transform.

Then, it is possible to determine, in the time domain, an approximation of the open loop top riser trajectory, that is a function of the reference trajectory for the riser bottom. The first step is the change of variable $l = 2\sqrt{z/g}$, which yields $\frac{\partial}{\partial z} = \frac{2}{gl}\frac{\partial}{\partial l}$, that transforms equation (4) into:

$$-l\frac{\partial^2\Upsilon}{\partial t^2}(l,t) - \tau l\frac{\partial\Upsilon}{\partial t}(l,t) + \frac{\partial\Upsilon}{\partial l}(l,t) + l\frac{\partial^2\Upsilon}{\partial l^2}(l,t) = 0 \tag{5}$$

Using a $t-$Laplace transform and considering the cable totally stopped at $t = 0$, equation (5) can be rewritten, with $\hat{\Upsilon}$ the Laplace transform of Υ, as:

$$-ls^2\hat{\Upsilon}(l,s) - \tau ls\hat{\Upsilon}(l,s) + \frac{\partial\hat{\Upsilon}}{\partial l}(l,s) + l\frac{\partial^2\hat{\Upsilon}}{\partial l^2}(l,s) = 0 \tag{6}$$

The change of variable $\zeta = il\sqrt{s(s+\tau)}$ transforms (6) into a Bessel equation of the first kind:

$$\zeta\hat{\Upsilon}(\zeta,s) + \frac{\partial\hat{\Upsilon}}{\partial\zeta}(\zeta,s) + \zeta\frac{\partial^2\hat{\Upsilon}}{\partial\zeta^2}(\zeta,s) = 0$$

The solution $\hat{\Upsilon}(z,s)$ has the following form:

$$\hat{\Upsilon}(z,s) = c_1 J_0(2i\sqrt{s(s+\tau)}\sqrt{z/g}) + c_2 Y_0(2i\sqrt{s(s+\tau)}\sqrt{z/g})$$

where J_0 and Y_0 are respectively the Bessel functions of first and second kinds [6]. Sought after solutions are finite for $\zeta = 0$, so these solutions must be such that $c_2 = 0$:

$$\hat{\Upsilon}(z,s) = \hat{\Upsilon}(0,s)J_0(2i\sqrt{s(s+\tau)}\sqrt{z/g})$$

Another way to define $\hat{\Upsilon}(z,s)$ is:

$$\hat{\Upsilon}(z,s) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\exp(-2\sqrt{s(s+\tau)}\sqrt{z/g}\sin\theta)\hat{\Upsilon}(0,s)d\theta \tag{7}$$

The term $\exp(-2\sqrt{s(s+\tau)}\sqrt{z/g}\sin\theta)$ in equation (7) can be expanded into a Taylor series around $\tau = 0$:

$$\begin{aligned}
&\exp(-2\sqrt{s(s+\tau)}\sqrt{\frac{z}{g}}\sin\theta) = \exp(-2\sqrt{s^2}\sqrt{z/g}\sin\theta)\cdots \\
&\cdots - \tau\exp(-2\sqrt{s^2}\sqrt{z/g}\sin\theta)\sqrt{z/g}\sin\theta\cdots \\
&\cdots + \tau^2\exp(-2\sqrt{s^2}\sqrt{z/g}\sin\theta)\left(\frac{z\sin^2\theta}{g} + \sqrt{z/g}\frac{\sin\theta}{2s}\right) + \cdots
\end{aligned}$$

The interest of the Taylor series is to simplify the Laplace transform inversion. The terms of the series can be analyzed as delays and easily associated to the reference trajectory $\Upsilon(0,t)$:

$$\begin{aligned}
\Upsilon(z,t) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \quad &\Big(\Upsilon(0,t-2\sqrt{z/g}\sin\theta) \\
&-\tau\Upsilon(0,t-2\sqrt{z/g}\sin\theta)\sqrt{z/g}\sin\theta \\
&+\tau^2\Upsilon(0,t-2\sqrt{z/g}\sin\theta)\frac{z\sin^2\theta}{g} \\
&+\frac{\tau^2}{2}\int_0^t\Upsilon(0,\varrho-2\sqrt{z/g}\sin\theta)\sqrt{z/g}\sin\theta d\varrho + \cdots\Big)\,d\theta
\end{aligned} \tag{8}$$

The open loop solution is found by numerically integrating equation (8), to define the control trajectory $\Upsilon_o(L,t)$, where L is the riser length. The simulation example in Figure 1 shows that the approximations made have a small effect on the system response. The considered discrete system for all numerical simulations comes from the discretization of equation (1), for a vertical 2 km long steel riser with (external diameter = 0.55 m, internal diameter = 0.5 m). These are typical values for a drilling riser in deep water. In figure 1, the hydrodynamic force is represented by equation (2).

3 System control

3.1 Unperturbed case

Consider $\Upsilon_R = \Upsilon - \Upsilon_o$ the relative displacement of Υ around the reference trajectory Υ_o. The objective is to define a tracking system to force the convergence of Υ_R to zero. Following the idea proposed by Thull et al [3], a candidate Lyapunov function, based on the system energy associated to Υ_R, is given by

$$V = \frac{L\Upsilon_R^2(L,t)}{2\vartheta^2} + \frac{1}{2}\int_0^L \left(z\left(\frac{\partial \Upsilon_R}{\partial z}\right)^2 + \left(\frac{\partial \Upsilon_R}{\partial t}\right)^2 \right) dz$$

Parameter ϑ represents the convergence time and determines the energy associated to the relative displacement of the structure top. Using equation (4), the time derivative of V is computed as follows:

$$\begin{aligned}\frac{dV}{dt} &= \frac{L\Upsilon_R(L,t)}{\vartheta^2}\frac{\partial \Upsilon_R}{\partial t}(L,t) + \int_0^L \left(z\frac{\partial \Upsilon_R}{\partial z}\frac{\partial^2 \Upsilon_R}{\partial z \partial t}\right) dz \cdots \\ &\cdots + \int_0^L \left(\frac{\partial \Upsilon_R}{\partial t}\left(\frac{\partial}{\partial z}\left(gz\frac{\partial \Upsilon}{\partial z}\right) - \tau\frac{\partial \Upsilon}{\partial t}\right)\right) dz\end{aligned} \tag{9}$$

After integration by parts, it comes

$$\frac{dV}{dt} = L\frac{\partial \Upsilon_R}{\partial t}(L,t)\left(g\frac{\partial \Upsilon_R}{\partial z}(L,t) + \frac{\Upsilon_R(L,t)}{\vartheta^2}\right) - \tau\int_0^L \left(\frac{\partial \Upsilon_R}{\partial t}\right)^2 dz$$

A proposed control law is

$$\frac{\partial \Upsilon_R}{\partial t}(L,t) = -\alpha\left(g\frac{\partial \Upsilon_R}{\partial z}(L,t) + \frac{\Upsilon_R(L,t)}{\vartheta^2}\right) \tag{10}$$

With this law, $dV/dt \leq 0$, so the system converges to the largest invariant set contained in $dV/dt = 0$. This set is such that

$$g\frac{\partial \Upsilon_R}{\partial z}(L,t) + \frac{\Upsilon_R(L,t)}{\vartheta^2} = 0$$

and

$$\int_0^L \left(\frac{\partial \Upsilon_R}{\partial t}\right)^2 dz = 0$$

this last relation implying $\partial\Upsilon_R/\partial t(z,t) = 0$ for all z and all t. For this set, by construction of the control law, we also have $\partial\Upsilon_R/\partial t(L,t) = 0$, which means that $\Upsilon_R(L,t)$ is constant. Then, $\partial\Upsilon_R/\partial z(L,t)$ is also constant. At rest, the balance of the external forces is given, at the top, by the sum of F_t, accounting for the horizontal part of the tension at the top of the structure, and F_p, the resultant of the disturbances. By definition, F_t is proportional to $\partial\Upsilon_R/\partial z(L,t)$. In this unperturbed case, $F_p = 0$, so $F_t = 0$ and $\partial\Upsilon_R/\partial z(L,t) = 0$, which leads to $\Upsilon_R(L,t) = 0$. This reasoning on external forces can be applied to the rest of the structure, and in particular gives $\Upsilon_R(0,t) = \Upsilon_R(L,t) = 0$: the re-entry operation is successful. Using the top position as the control variable, the control law writes in practice:

$$\Upsilon(L,t) = \Upsilon_o(L,t) - \alpha \int_0^t \left(g\frac{\partial \Upsilon_R}{\partial z}(L,v) + \frac{\Upsilon_R(L,v)}{\vartheta^2}\right) dv \tag{11}$$

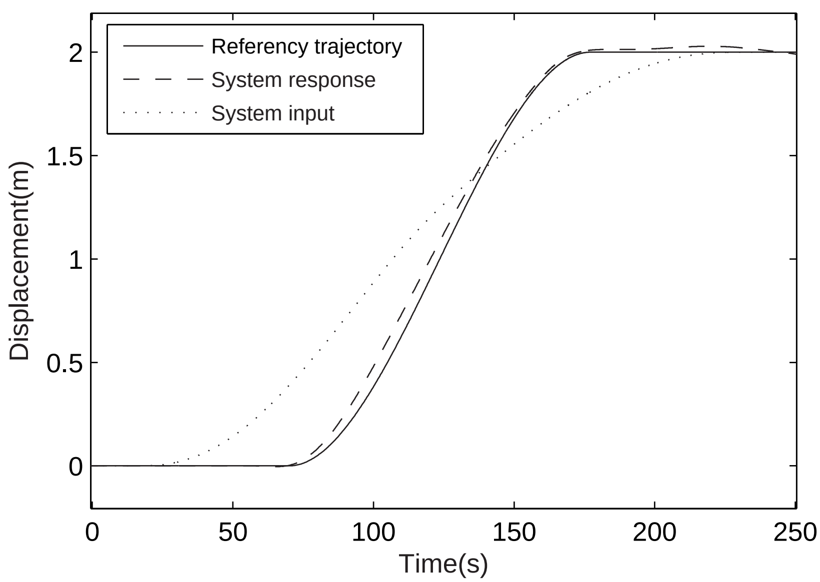

Figure 1: Unperturbed case. Reference trajectory and system response.

3.2 Disturbances

In practical cases of interest, the structure has two main kinds of disturbances, that change the flow speed: waves and marine currents. The waves have their energy concentrated on the first meters of depth. They normally have two mains frequencies, the faster around 50 mHz and the other around 10 mHz. The marine currents have their energy distributed more uniformly, with smooth time variations. The effect of the marine currents generates an offset that slowly changes. The changes of the flow speed due to these disturbances are represented by the function $U(z,t)$, that, with respect to the unperturbed case given by (2), induces the following changes in the form of the hydrodynamic forces (m_I is a constant usually called the inertia coefficient):

$$F(z,t) = -m_F\frac{\partial^2\Upsilon}{\partial t^2} + \mu\left(U(z,t) - \frac{\partial\Upsilon}{\partial t}\right)\left|U(z,t) - \frac{\partial\Upsilon}{\partial t}\right| + m_I\frac{\partial U}{\partial t} \tag{12}$$

Constant disturbances During the re-entry operation, the marine current can be assumed constant and generates a static deformation of the structure $\bar{\Upsilon}(z)$. If the assumption of small angles in the vertical direction holds, $\bar{\Upsilon}(z)$ can be defined by

$$-\frac{\partial}{\partial z}\left(gz\frac{\partial\bar{\Upsilon}}{\partial z}(z)\right)=\frac{\mu}{m}U(z)|U(z)|$$

with the boundary condition $\bar{\Upsilon}(L)=0$. In these conditions, when the control law used in the unperturbed case is applied, it is easily proved that the system is stabilized at an equilibrium point given by $\Upsilon(z,t)=\bar{\Upsilon}(z)-g\vartheta^2\partial\bar{\Upsilon}/\partial z(L)$. To avoid this bias at the bottom, we propose the following solution. Before applying the control law:

- Estimate the current distance from the bottom to the wellhead. From this estimation, choose a reference trajectory for the bottom, and compute the reference trajectory for the top as in the unperturbed case.

- Estimate the current angle at the top $(\partial\bar{\Upsilon}/\partial z(L))$.

The definition of $\Upsilon_R(z,t)$ is modified into $\Upsilon_R(z,t)=\Upsilon(z,t)-\Upsilon_o(z,t)-\bar{\Upsilon}(z)$. Then, the computation of the control law obeys (11), exactly like in the unperturbed case. Indeed, the only difference lies in the use of the estimation $\partial\bar{\Upsilon}/\partial z(L)$. Simulation closed loop results are presented in Figure 2.

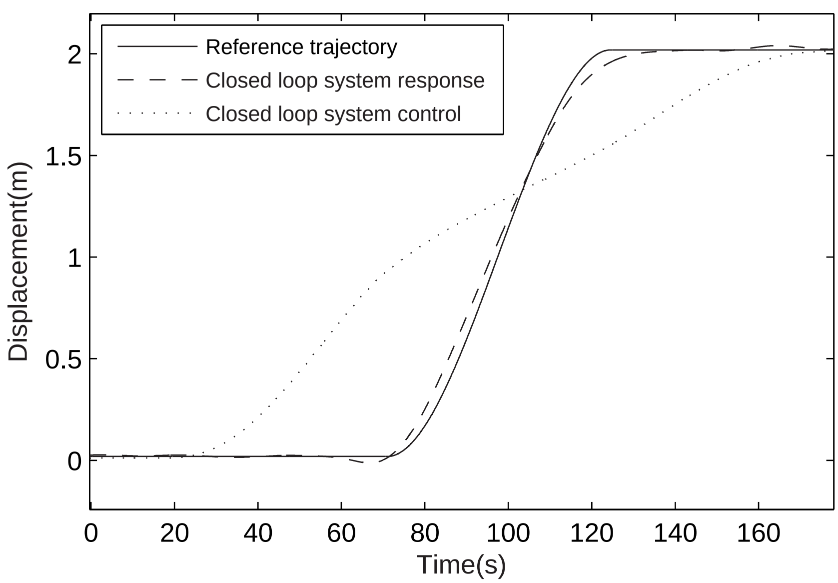

Figure 2: Structure under constant disturbance due to the current. Reference trajectory and closed loop response.

Periodic disturbances In the case of waves, the structure deformation linked to this disturbance is not constant. We still miss a formal approach for the closed loop control in this case. Approaches like those proposed by Sabri can be used

[4]. However, regarding equation (12), it is possible to observe that an artificial increase of the structure speed $\partial\Upsilon/\partial t$ implies a larger system damping, that reduces the relative effect of the flow speed changes. The proposed approach consists in simply using an open loop trajectory, that is fast enough to increase the damping during a given period of time, in order to reduce the effect of waves. Figure 3 provides an example of what can be obtained with this approach. The shape of the reference trajectory is such that the reference speed is kept large almost until the end of the bottom displacement. This avoids the increase of the disturbances before the structure has reached its target. A problem with this approach is that, for a given disturbance, a maximum speed must be used, during a large enough period for the disturbances to attenuate. This turns out to require a minimum initial distance between the bottom of the structure and the wellhead. The choice of this maximum speed is constrained by the following considerations, that are difficult to quantify a priori and do not preclude the constraints of the actuators:

- if the speed is too large, the small angles assumption does not hold anymore, and more terms are required in the expansion of the damping term;
- if the acceleration is too large, the beam effect is no more negligible.

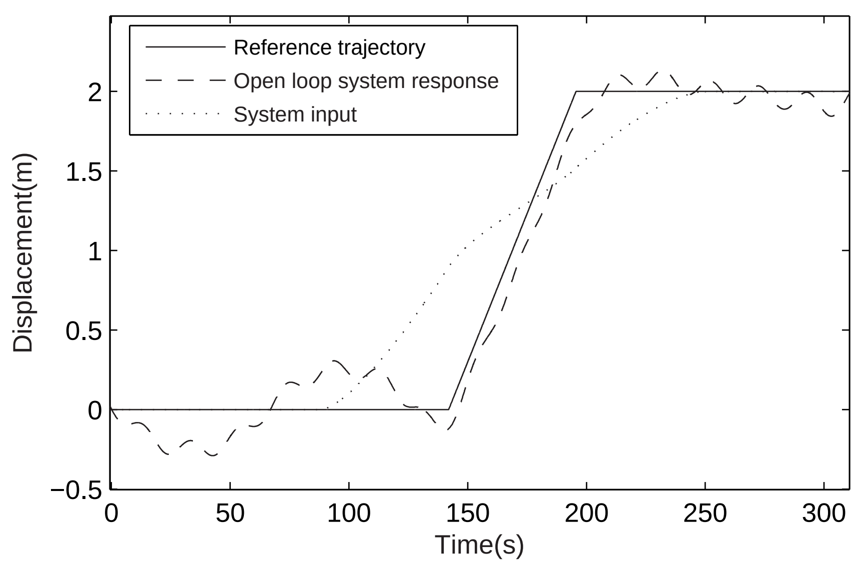

Figure 3: Structure under periodic disturbance due to waves. Reference trajectory and open loop response.

Conclusion

This article presents an analytical solution for the trajectory planning of a damped cable and uses a tracking system able to correct systems imperfections and constant disturbances. Another point is the possibility to reduce disturbances with the design of the open loop trajectory. However the design of a closed loop to reject periodic disturbances remains an open problem.

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