Homogeneous observers with dynamic high gains

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HOMOGENEOUS OBSERVERS WITH DYNAMIC HIGH GAINS

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Abstract: We propose an extension of the well-known high-gain observer design by incorporating an update law for the gain as well as higher order output error terms in the correction. This extension is obtained by applying techniques of dynamic scaling and homogeneity in the bi-limit. This allows a wider class of systems in feedback form to be dealt with. Furthermore, the gains of the observer obtained are adaptated to the local incremental rate of the nonlinearities.

Keywords: High-gain observers, Homogeneity in the bi-limit, Dynamic scaling.

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1. INTRODUCTION

The most systematic answer proposed to solve a problem of state observation for non-linear systems, is very likely a high gain observer (see (Gauthier et al., 1992; Gauthier and Kupka, 2001) and the references therein). We extend it in two directions : homogeneity and gain adaptation. A motivation for invoking these two techniques comes from considering the following simple system :

$$\dot{x}_1 = x_2 \ , \quad \dot{x}_2 = f_2(x_1, x_2, u) \ , \quad y = x_1 \ , \qquad (1)$$

with

$$f_2(x_1, x_2, u) \ = \ g(x_1)\, x_2 \ + \ x_2^{1+p} \ + \ u \ ,$$

where $p \geq 0$ is a real number, g is a locally Lipschitz function and u is a known input.

When $p = 0$, the non-linearity satisfies :

$$|f_2(x_1, x_2, u) - f_2(x_1, \hat{x}_2, u)| \qquad (2)$$
$$\leq \ |g(x_1) + 1|\, |x_2 - \hat{x}_2| \ .$$

The term $|g(x_1) + 1|$ appears as a bound on the incremental rate of the non-linearity and depends on the output. This class of nonlinearities has already been studied in the context of stabilization by output feedback in (Praly, 2003) (see also (Krishnamurthy et al., 2003)) and we know that, despite this system is not globally Lipschitz, a high gain observer can be used but with a gain updated from a output dependant bound on the incremental rate.

When p is in $(0, 1)$, inequality (2) becomes :

$$|f_2(x_1, x_2, u) \ - \ f_2(x_1, \hat{x}_2, u)| \qquad (3)$$
$$\leq \ (|g(x_1)| + (1+p)|\hat{x}_2|^p)\, |x_2 - \hat{x}_2|$$
$$+ \ |x_2 - \hat{x}_2|^{1+p} \ .$$

The first term in the right hand side yields the bound $|g(x_1)| \ + \ (1 + p)\, |\hat{x}_2|^p$ on the local incremental rate. It can be handled in a way similar to the one in (2), although it depends now also on $\hat{x}_2$. The second term, $|x_2 - \hat{x}_2|^{1+p}$ is a rational power

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of the norm of the error $|x_2 - \hat{x}_2|$. To deal with this term we use the homogeneous in the bi-limit observer introduced in (Andrieu at al., 2006).

In the following we address the problem of state observation for systems whose dynamics admit a global explicit observability canonical form (Gauthier and Kupka, 2001, Equation (20)) and in which the nonlinearities have incremental growths bounded as in (3) and therefore may be not globally Lipschitz. However we restrict our attention to estimating the state only for solutions which remain bounded in positive time.

One interest of our new observer lies in the fact that we try to fit the nonlinearities better than what is done in the usual high gain observer. Namely, instead of a simple linear term, the effects of the nonlinearities are captured by a linear term with a solution dependent gain plus a rational power term. From this we expect the possibility of achieving better performance. We illustrate this more practical aspect via the analysis of an academic model of a bioreactor. In particular, we show via simulations the improvement which can be obtained.

In section 2 the main theoretical result of the paper is stated and discussed. It is illustrated in Section 3. Unfortunately, due to space limitations, we cannot give the proof of this result. It can be found in (Andrieu at al., 2007).

Notation : For any real number r, we define the function $w \in \mathbb{R} \mapsto w^r$ as $w^r = \text{sign}(w)\,|w|^r$. For instance, to recover the quadratic function we must write $|x^2|$ or $|x|^2$.

2. MAIN THEORETICAL RESULT

We consider systems whose dynamics can be approximated by a global explicit observability canonical form, i.e. there are globally defined coordinates $(x_1, \ldots, x_n)$ in $\mathbb{R}^n$ such that we have [1] :

$$\left\{\begin{array}{rl} \dot{x}_1 &= f_1(u,y) + a_1(y)\,x_2 + \delta_1(t) \ , \\ \vdots & \\ \dot{x}_i &= f_i(u,y,x_2,\ldots,x_i) + a_i(y)\,x_{i+1} + \delta_i(t) \ , \\ \vdots & \\ \dot{x}_n &= f_n(u,y,x_2,\ldots,x_n) + \delta_n(t) \ , \\ y &= x_1 + \delta_y(t) \ , \end{array}\right. \qquad (4)$$

where y is the measured output in $\mathbb{R}$ and the functions a_i and f_i are locally Lipschitz. u is a compact notation for representing known inputs and a finite number of their derivatives. The vector $\delta = (\delta_1, \ldots, \delta_n)$ represents unknown inputs and in particular the effects unmodeled by the model dynamics, and δ_y is a measurement noise.

To simplify the following equations we denote by $\mathcal{S}$ the left shift matrix of order n, i.e. $\mathcal{S}\,x = (x_2, \ldots, x_n, 0)^T$ and we let :

$$f(u,y,x) = (f_1(u,y,x), \ldots, f_n(u,y,x)) \ ,$$
$$A(y) = \text{diag}(a_1(y), \ldots, a_n(y)) \ .$$

Theorem 1. Consider system (4). Suppose there exist a continuous function $\mathfrak{a}$ satisfying :

$$0 < \rho \leq \mathfrak{a}(y) \ , \quad 0 < \underline{\mathfrak{A}} \leq \frac{a_j(y)}{\mathfrak{a}(y)} \leq \overline{\mathfrak{A}} \quad \forall y \in \mathbb{R} \quad (5)$$

for $j \in (1, n)$, a real number d_∞ in $[0, \frac{1}{n-1})$, a positive real number c_∞, a continuous function Γ and real numbers v_j in $[0, \frac{1}{j-1})$, for $j = 2, \ldots n$, such that, for all i in $\{2, \ldots, n\}$ and all $(\hat{x}, x, y, u)$ in $\mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^m$, we have :

$$\begin{aligned} &|f_i(u,y,\hat{x}_2,\ldots,\hat{x}_i) - f_i(u,y,x_2,\ldots,x_i)| \qquad (6) \\ &\quad \leq \Gamma(u,y)\left(1 + \sum_{j=2}^{n} |\hat{x}_j|^{v_j}\right) \sum_{j=2}^{i} |\hat{x}_j - x_j| \\ &\qquad + c_\infty \sum_{j=2}^{i} |\hat{x}_j - x_j|^{\frac{1-d_\infty(n-i-1)}{1-d_\infty(n-j)}} \ . \end{aligned}$$

Then for all sufficiently small strictly positive real numbers b there exists a function K such that, for all sufficiently small strictly positive real number φ_1 and sufficiently large real numbers φ_2 and φ_3, we can find functions β_W and β_L of class $\mathcal{KL}$ and functions γ_W and γ_L of class $\mathcal{K}$ such that the observer

$$\begin{aligned} \dot{\hat{x}} &= A(y)\mathcal{S}\hat{x} + f(u,y,\hat{x}) \\ &\qquad + L\,\mathfrak{L}\,A(y)\,K\left(\frac{\hat{x}_1 - y}{L^b}\right) \qquad (7) \\ \dot{L} &= L\left[\varphi_1(\varphi_2 - L) + \varphi_3\,\Omega(u,y,\hat{x})\right] \ , \qquad (8) \end{aligned}$$

where $\mathfrak{L} = \text{diag}(L^b, \ldots, L^{n+b-1})$, initialized with $L(0) \geq \varphi_2$, has the following properties.

For each solution $t \mapsto x(t)$ of (4) right maximally defined on $[0, \mathcal{T})$, the observer solution is defined on the same interval and the error estimate $\mathfrak{e} = \hat{x} - x$ satisfies, for all t in $[0, \mathcal{T})$,

$$\begin{aligned} |\mathfrak{L}(t)^{-1}\mathfrak{e}(t)| &\leq \beta_W\left(\mathfrak{L}(0)^{-1}\mathfrak{e}(0), t\right) \qquad (9) \\ &\quad + \sup_{s\in[0,t]} \gamma_W\left(\left|\left(\begin{array}{c} \frac{\delta(s)}{\varphi_2} \\ \frac{\mathfrak{a}(y(s))\delta_y(s)}{\rho} \end{array}\right)\right|\right) \end{aligned}$$

where L satisfies, for all t in $[0, \mathcal{T})$,

$$L(t) \leq 4\varphi_2 + \beta_L\left(\left(\begin{array}{c}\mathfrak{e}(0) \\ L(0)\end{array}\right), t\right) \qquad (10)$$

[1] To facilitate the analysis y is used as argument in the dynamics. As a consequence δ may not be zero in case of exact modelling only due to measurement noise.

$$+ \sup_{s\in[0,t]} \gamma_L \left\| \begin{pmatrix} \dfrac{\delta(s)}{\varphi_2} \\ \dfrac{\mathfrak{a}(y(s))\delta_y(s)}{\rho} \\ \Gamma(u(s),y(s)) \\ x(s) \end{pmatrix} \right\| .$$

2.1 Discussion on the assumptions

The functions a_i and f_i in (4) are not uniquely defined. They can be modified by changing coordinates and, in this way, possibly satisfy conditions (6). For instance we can replace x_2 by $\mathscr{x}_2 = \check{f}_1(u,y) + \check{a}_1(y)x_2$ where $\check{f}_1$ and $\check{a}_1 > 0$ are arbitrary. And so on for $\mathscr{x}_i$ replacing x_i.

With the form (4), the main assumption of Theorem 1 is the inequality (6). In essence, it imposes that the function

$$\Omega(u,y,\hat{x}) \ = \ \Gamma(u,y) \left(1 + \sum_{j=2}^{n} |\hat{x}_j|^{v_j}\right) ,$$

is a bound on the local incremental rate. It is also setting a fractional power restriction, $\frac{1-d_\infty(n-i-1)}{1-d_\infty(n-j)}$, with d_∞ in $[0, \frac{1}{n-1})$ on the function increment for large argument increments. A motivation for this rational power is that, by following arguments similar to those used in (Mazenc et al., 1994), it can be proved that, for $d_\infty > \frac{1}{n-1}$, there is no continuous function K such that the origin of the following system is globally asymptotically stable

$$\dot{\mathfrak{e}} \ = \ \mathcal{S}\,\mathfrak{e} + (\,0\ \dots\ 0\ 1\,)^T\, |\mathfrak{e}_n|^{1+d_\infty} + K(\mathfrak{e}_1) \ .$$

We end this discussion by considering the system (1). With $d_\infty = p$, we get that inequality (3) is in the form (6) with $\Gamma(u,y) = (|g(y)| + (1+p))$ and $v_2 = p$. Hence, Theorem 1 applies to system (1) when p is in the interval $[0,1)$. It is interesting to observe that when $p > 1$ and $u = 0$, there do not exist any observer guaranteeing convergence of the estimation error within the domain of existence of the solutions (see (Astolfi and Praly, 2006, Proposition 1)).

2.2 Discussion on the result

Because of the presence of $\sup_s |x(s)|$, Theorem 1 says in particular that the observer (7),(8) gives, at least for bounded solutions, an estimation error converging to a ball centered at the origin and with radius depending on the L^∞-norm of the disturbances δ and δ_y.

Since we restrict our attention to bounded solutions, the reader may think that we are back to the global Lipschitz case. This is not completely true since in this case the "Lipschitz constant" is solution dependent and therefore unknown in advance for designing the observer. It has to be learned online and this is what L is doing in (8). The update law for L is very similar to the one introduced in (Praly, 2003) (see also (Krishnamurthy et al., 2003)). The difference is in the fact that (8) depends also on $\hat{x}$ and u and not only on y. It is because of this dependence on $\hat{x}$ that we need to put restrictions on the v_j.

The update law (8) is such that, if Ω were differentiable along the solutions, it would give :

$$\overbrace{L - \left(\varphi_2 + \frac{\varphi_3}{\varphi_1}\Omega\right)}^{\cdot} = \frac{\varphi_3}{\varphi_1}\dot{\Omega} - \varphi_1 \left[L - \left(\varphi_2 + \tfrac{\varphi_3}{\varphi_1}\Omega\right)\right] \ .$$

This says that L would track $\varphi_2 + \frac{\varphi_3}{\varphi_1}\Omega$ up to an error related to the magnitude of $\dot{\Omega}$.

Equation (8) is very different from update laws where the estimation error $\hat{x}_1 - y$ is used to lead the adaptation as we can find in (Bullinger and Allgöwer, 2005; Lei et al., 2005) for instance. The latter leads typically to a gain which, along closed loop solutions, is a nondecreasing function of time. This renders its interest in practice much reduced and, even worse, it is well known and analyzed that it leads to nonrobust behaviour.

Here instead, by equation (10), we are guaranteed that the updated gain L remains bounded for any bounded system solution even in the presence of modelling errors δ and measurement noise δ_y.

2.3 Comparison with published results known to the authors

Theorem 1 belongs to the family of results relying on a domination approach where the specificities of the nonlinearities are not exploited besides the fact that they can be dominated in some way. In the following we restrict our attention to results in the same family we are aware of.

High gain (linear) observers have a long history. The prototype result is (Gauthier and Kupka, 2001, Theorem 6.2.2). It deals with systems admitting an observability canonical representation more general than (4) by being implicit in x_{i+1}. But the domination in (6) is given only by $\Gamma\ \sum_{j=2}^{i} |\hat{x}_j - x_j|$ with Γ constant.

The case where Γ may depend on y, and actually also on u, can be deduced from (Praly, 2003) when the a_i's are constant and from (Krishnamurthy et al., 2003) when the a_i's are y-dependent. As we have seen above, this extension is made possible by introducing an update law in the form of (8).

The idea of homogeneous correction terms has been indroduced in (Qian, 2005) but with the objective of incorporating them in an output feedback scheme. Hence, the observer was designed only for a pure chain of integrators, i.e. when the a_i's are constant and the f_i's are zero and it is homogeneous in the classical weighted sense.

An other observer is proposed in (Lei et al., 2005) for systems with bounded solutions and admitting the same form (4), with the a_i's constant and $f_1 = \ldots = f_{n-1} = 0$, but with no restriction at all on f_n. However this is obtained by having a gain which grows monotonically with time along the solutions, with all the potential problems we have mentioned above.

In (Astolfi and Praly, 2006), the same system as in (Lei et al., 2005) is considered. Instead of restricting the analysis to bounded solutions, the existence of a state norm observer is assumed. Then convergence of the estimation error is obtained within the domain of existence of the solutions. Unfortunately, for the time being, this contribution remains mainly at the conceptual level in view of the difficulty in constructing the state norm observer.

3. DISCUSSION AND EXAMPLE

To illustrate the interest for applications of the observer we propose, we consider the same "academic" bioreactor as the one studied in (Gauthier et al., 1992), where the classical high-gain observer has been introduced and fully analyzed. The dynamics are described by a Contois model which, in normalized variables and time, is :

$$\left\{\begin{array}{rcl} \dot{\eta}_1 &=& \dfrac{\eta_1\eta_2}{\hbar\eta_1+\eta_2} - u\,\eta_1 \ , \\ \dot{\eta}_2 &=& -\dfrac{\eta_1\eta_2}{\hbar\eta_1+\eta_2} + u\,(1-\eta_2) \ , \\ y &=& \eta_1 \end{array}\right. \tag{11}$$

The parameter $\hbar$ is a positive real number and the control input u is in the interval $\mathcal{M}_u = [u_{\min}, u_{\max}] \subset (0,1)$. In (Gauthier et al., 1992), it is observed that the set :

$$\mathcal{M}_\eta = \{(\eta_1,\eta_2) \in \mathbb{R}^2 \ : \ \eta_1 \geq \epsilon_1,\ \eta_2 \geq \epsilon_2,\ \eta_1+\eta_2 \leq 1\} \ ,$$

where, $\epsilon_1 = \frac{(1-u_{\max})\epsilon_2}{\hbar u_{\max}}$, and $u_{\min} \geq \frac{\epsilon_2}{\hbar(1-\epsilon_2)+\epsilon_2}$ is forward invariant. This important remark guarantees that the bioreactor state solutions are bounded and actually remain in a known compact set. Following (Gauthier et al., 1992), we change the coordinates as :

$$\begin{array}{rcl} \mathcal{M}_\eta & \mapsto & \mathcal{M}_x \ = \ F(\mathcal{M}_\eta) \ , \\ (\eta_1,\eta_2) & \mapsto & (x_1,x_2) = \left(\eta_1, \dfrac{\eta_1\eta_2}{\hbar\eta_1+\eta_2}\right) \ . \end{array}$$

In these new coordinates the system is in the explicit observability canonical form :

$$\dot{x}_1 = x_2 - u\,x_1 \ , \quad \dot{x}_2 = f_2(x_1,x_2,u) \ , \quad y = \eta_1 \ ,$$

with,

$$f_2(x_1,x_2,u) = m_0 + m_1x_2 + m_2x_2^2 + m_3x_2^3 \tag{12}$$

where :

$$m_0 = \frac{u}{\hbar} \ , \qquad m_1 = -u - \frac{1}{\hbar} - \frac{2\,u}{\hbar x_1} \ ,$$
$$m_2 = \frac{2}{\hbar x_1} + \frac{u}{\hbar x_1^2} \ , \qquad m_3 = \frac{\hbar - 1}{\hbar x_1^2} \ .$$

Note that for all (x_1,x_2,u) in $\mathcal{M}_x \times \mathcal{M}_u$, we have : $\underline{x}_2(x_1) = x_1\,\frac{\epsilon_2}{\hbar x_1+\epsilon_2} \leq x_2 \leq x_1\,\frac{1-x_1}{1-x_1+\hbar x_1} = \overline{x}_2(x_1)$. Hence, for a given (u,x_1) in $[u_{\min},u_{\max}] \times [\epsilon_1, 1-\epsilon_2]$, x_2 is in the interval $[\underline{x}_2(x_1), \overline{x}_2(x_1)]$ and, without loss of generality, to evaluate f_2 in (12), we can replace (x_1,x_2) by (x_{1s},x_{2s}) defined as

$$\begin{array}{rcl} x_{1s} &=& \max\{\epsilon_1, \min\{1-\epsilon_2, x_1\}\} \ , \\ x_{2s} &=& \max\{\underline{x}_2(x_{1s}), \min\{\overline{x}_2(x_{1s}), x_2\}\} \end{array}$$

and therefore assume that f_2 is globally Lipschitz in (x_1,x_2).

To design an observer by following a domination approach we have to start by choosing a bound for the function increment $|f_2(x_1,x_2,u) - f_2(x_1,\hat{x}_2,u)|$.

For a nominal high gain observer, as in (Gauthier et al., 1992), the bound is, for all (x_1,x_2) and $(x_1,\hat{x}_2)$ in $\mathcal{M}_x$ and all u in $\mathcal{M}_u$,

$$|f_2(x_1,x_2,u) - f_2(x_1,\hat{x}_2,u)| \ \leq \ df_{2\max}\,|x_2-\hat{x}_2| \ .$$

where from the Mean Value Theorem,

$$df_{2\max} = \max_{(u,x_1,x_2)\in\mathcal{M}_u\times\mathcal{M}_x} |m_1 + 2m_2x_2 + 3m_3x_2^2| \ .$$

For a high gain observer with updated gain, the bound is :

$$|f_2(x_1,x_2,u) - f_2(x_1,\hat{x}_2,u)| \leq \Omega_1(u,x_1,\hat{x}_2)\,|x_2-\hat{x}_2| \ ,$$

with

$$\Omega_1(u,x_1,\hat{x}_2) = \max_{x_2\in[\underline{x}_2(x_{1s}),\overline{x}_2(x_{1s})]} |m_1 + m_2(\hat{x}_2+x_2) + m_3[\hat{x}_2^2 + \hat{x}_2x_2 + x_2^2]| \ .$$

Since $\hat{x}_2$ is an estimate of x_2 which remains in $[\underline{x}_2(x_{1s}), \overline{x}_2(x_{1s})]$, as argument of Ω_1, we can replace, $\hat{x}_2$ by

$$\hat{x}_{2s} = \max\{\underline{x}_2(x_{1s}), \min\{\overline{x}_2(x_{1s}), \hat{x}_2\}\} \ .$$

It follows that Theorem 1 applies with $d_\infty = 0$.

Finally for our observer with both updated gain and rational power error term, the bound is :

$$|f_2(x_1,x_2,u) - f_2(x_1,\hat{x}_2,u)| \leq \Omega_2(u,x_1,\hat{x}_2)\,|x_2-\hat{x}_2| \ + \ c_\infty|x_2-\hat{x}_2|^{1+p} \ ,$$

with p in $(0,1)$ and where

$$\Omega_2(u,x_1,\hat{x}_2) = \max_{x_2\in[\underline{x}_2(x_{1s}),\overline{x}_2(x_{1s})]}$$
$$|m_1 + \hat{x}_2^p([m_2+m_3\hat{x}_2][\hat{x}_2^{1-p}+x_2^{1-p}] + m_3x_2^{2-p})])|$$

and

$$c_\infty = \max_{(u,x_1,x_2,\hat{x}_2)\in\mathcal{M}_u\times\mathcal{M}_x\times[\underline{x}_2(\epsilon_1),\overline{x}_2(1-\epsilon_2)]}$$
$$|(m_2+m_3\hat{x}_2)x_2^{1-p} + m_3x_2^{2-p}|$$

In this case, Theorem 1 gives the following observer :

$$\begin{cases} \dot{\hat{x}}_1 &= \hat{x}_2 - u\,y - L^{1+b}\,k_1\left(\dfrac{[\hat{x}_1-y_s]}{L^b}\right) , \\ \dot{\hat{x}}_2 &= f_2(y_s,\hat{x}_{2s},u) \\ & \quad - L^{2+b}\,k_2\left(\ell\,k_1\left(\dfrac{[\hat{x}_1-y_s]}{L^b}\right)\right) , \\ \begin{pmatrix}\hat{\eta}_1\\ \hat{\eta}_2\end{pmatrix} &= F^{-1}\begin{pmatrix}\hat{x}_1\\ \hat{x}_2\end{pmatrix} , \\ \dot{L} &= L\left[\varphi_1\,(\varphi_2 - L) + \varphi_3\,\Omega_2(u,y_s,\hat{x}_{2s})\right] , \end{cases}$$

where $y_s = \max\{\epsilon_1, \min\{1-\epsilon_2, y\}\}$,

$$k_1(s) = s + s^{\frac{1}{1-p}} \quad , \qquad k_2(s) = s + s^{1+p}$$

and b, φ_i and ℓ are parameters to be chosen.

Since we have, for all (x_1,x_2,u) in $\mathcal{M}_x\times\mathcal{M}_u$:

$$\frac{\partial f_2}{\partial x_2}(x_1,x_2,u) \le \Omega_2(u,x_1,x_2)$$
$$\le \Omega_1(u,x_1,x_2) \le df_{2\max}$$

we expect the high gain observer with updated gain to give better performance than the one without adaptation, and the new one proposed in this paper to give even better behavior in particular in the presence of measurement noise.

3.1 Simulations

To support our claims on the behavior of the observer, we present some simulations. They are only illustrations since we haven't "optimized" any observer parameter. In addition we do not claim that our observer is the best one for this application. In particular the system (11) being contracting, a simple copy (without correction term) gives an observer which is not sensitive to measurement noise.

The control input is selected as

$$\begin{aligned} u(t) &= 0.5 \quad \text{if } t < 10 , \\ &= 0.02 \text{ if } 10 \le t < 204, \\ &= 0.6 \quad \text{if } 20 \le t < 35 , \\ &= 0.1 \quad \text{if } 35 \le t . \end{aligned}$$

From this we have chosen $u_{\min} = 0.01$ and $u_{\max} = 0.7$ and ϵ_1 and ϵ_2 accordingly. Also, we have introduced two disturbances :

– the measurement disturbance is a Gaussian white noise with standard deviation equals to 10% of the η_1 domain $[\epsilon_1, 1-\epsilon_2]$, i.e. $= 0.05$.
– a 20% error in $\hbar$. The value used for the system (11) is 1, whereas the one in the observers is 0.8

For the homogeneous with updated gain observer we have used the following values

$$p = 0.9\ ,\ \ b = 0.5\ ,$$
$$\varphi_1 = 3\,,\ \varphi_2 = 0.01\,,\ \varphi_3 = 3\,,\ \ell = 3\ .$$

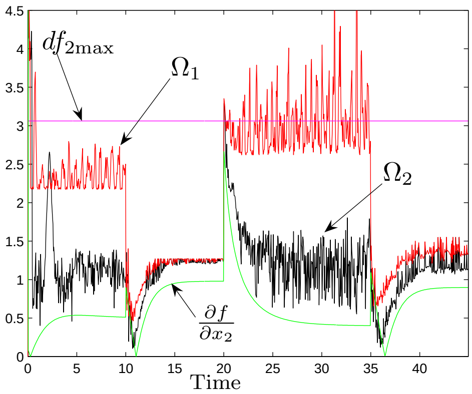

Fig. 1. Approximations of the local incremental rates in the 3 cases.

Figure 1 shows the values of the estimates of the local incremental rate of f_2 (i.e. $\frac{\partial f_2}{\partial x_2}$) along the solution, when we follow a classical high-gain approach (i.e. $df_{2\max}$), an updated high-gain approach (i.e. Ω_1), and a homogeneous updated high-gain approach (i.e. Ω_2).

We observe that the noise, present in the measure, is reflected in these estimations. Nevertheless the predicted order $df_{2\max} \ge \Omega_1 \ge \Omega_2 \ge \frac{\partial f_2}{\partial x_2}$ is observed in the mean.

Figure 2 dysplays the plot of η_2 and $\hat{\eta}_2$ given by the observer with constant gain deduced from $df_{2\max}$ (top), the observer with adapted gain deduced from Ω_1 (middle), our new observer with updated gain deduced from Ω_2 and homogeneity with $p = 0.9$ (bottom). We observe in the three cases a bias which is due to the error in $\hbar$ and increases with the estimates of the local incremental rate. We see also a strong correlation between the standard deviation of the error $\hat{\eta}_2 - \eta_2$ and the magnitude of these estimates respectively used, i.e. $df_{2\max}$, Ω_1 and Ω_2. As predicted the best result is given by the new observer based on Ω_2.

4. CONCLUSION

We have presented a modification of the classical high gain observer with the introduction of a gain updating mimicking the one of an extended Kalman filter and of a homogeneous in the bi-limit correction term. We have shown that this extends the domain of applicability by allowing some nonglobally Lipschitz nonlinearities. However the convergence result is established only for bounded solutions. We have also shown by means of an example, that the modification may

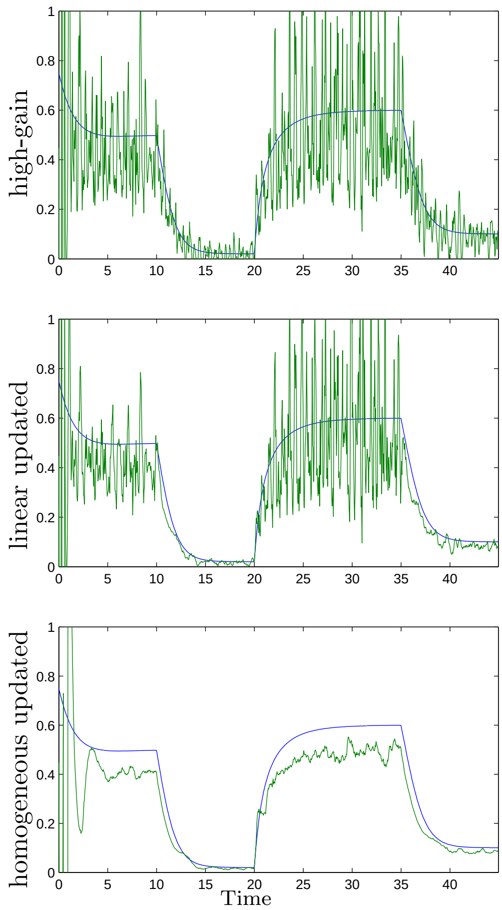

Fig. 2. Estimation of η_2 with the 3 observers

improve performance by allowing a better fit of the incremental rate of the nonlinearities.

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