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# Damping Behaviour of Vibrating Shape Memory Alloy Rods Investigated by a Novel Constitutive Model

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Hysteretic behaviour of shape memory alloys (SMAs) is highly important for design and applicability of these materials in active structural elements like rods. Especially the damping performance of SMAs depend strongly on their hysteretic characteristics. Experimental investigations show the influences of stress on the hysteretic cycle. The current study shows some computational results of a constitutive model which is capable to investigate the effect of an external applied stress field on the hysteretic cycle according to a recently developed method on the basis of statistical mechanics method.

## 1 INTRODUCTION

Smart materials especially shape memory alloys (SMAs) have become highly important during the last years. Nowadays there exist several applications in engineering [4]. As a consequence of the highly distinct hysteretic behavior SMAs are used in vibration control devices for civil structures [7]. The understanding of the temperature induced and stress assisted martensitic phase transformation is the key for explaining and modeling their constitutive behavior. [1]. The high damping capabilities of SMAs are a direct consequence of their hysteretic behavior.

## 2 THEORY AND MODEL

Based on statistical physics Oberaigner et.al [2, 6] developed a theory for a system of quasiparticles. Each of them stands for a single crystal and its thermal interactions. A polycrystal is built by a system of quasiparticles. From the canonical Gibbs-Potential of the polycrystal, all properties of interest can be derived. The Gibbs-Potential $G$ is given by the natural logarithm of the partition function which itself is a trace over Boltzmann weights of thermodynamical states. In case of a one variant model the partition function depends only on the interface Hamiltonians $H_{AM} \geq H_{AA} \geq H_{MM}$ and on the Hamiltonian $H_A$ of the austenitic phase and $H_M$ of the martensitic phase $H_M$. They are given by:

$$H_A(T,\underline{\sigma}) = F_A(T) - \frac{1}{2\eta_A}\underline{\sigma} : \underline{\underline{\underline{\underline{C}}}}^A : \underline{\sigma} = G_A(T,\underline{\sigma}) \,, \tag{1}$$

$$H_M(T,\underline{\sigma}) = F_M(T) - \frac{1}{2\eta_M}\underline{\sigma} : \underline{\underline{\underline{\underline{C}}}}^M : \underline{\sigma} - \frac{1}{\eta_M}\underline{\sigma} : \varepsilon^{\mathrm{tr}}_{\mathrm{cryst}} = G_M(T,\underline{\sigma}) \,. \tag{2}$$

The Hamiltonians are defined classically and are therefore functions of stress $\sigma$ and temperature $T$. In Eq.1 and Eq.2 the functions $F_A(T)$ and $F_M(T)$ are the free energies of the pure phases. They are computed in a thermodynamical consistent form from the heat capacity of a NiTiNb single crystal [8]. Furthermore the Hamiltonians are based on measurable properties of the pure phases like elastic moduli or compliancies $\underline{\underline{\underline{\underline{C}}}}^A, \underline{\underline{\underline{\underline{C}}}}^M$ molar densities $\eta_A, \eta_M$ and crystallographic transformation strain $\varepsilon^{\mathrm{tr}}_{\mathrm{cryst}}$.

## 3 RESULTS AND DISCUSSION

In Fig.1 to Fig.3 cycles for a temperature driven phase transformation at different external applied stresses levels (20 MPa to 160 MPa) are shown. As computed results the total strain (Fig.1), the martensitic phase fraction evolution (Fig.2) and the stress-strain loops (Fig.3) are presented. A three-dimensional representation of the temperature dependent stress-strain loops is given in Fig.4. The calculated results agree qualitatively well with experimental observations and as it is observed in experiments [3], the martensitic start temperature shifts to higher temperatures if the applied stress is increased. Based on the developed statistical physics theory the stress dependent hysteretic behavior of the NiTiNb alloy can be calculated which agrees with experimental observations qualitatively well. In dynamical applications the damping behavior of SMAs is characterized by the enclosed area of the hysteresis. It is well known from experiments that at higher stress levels the area

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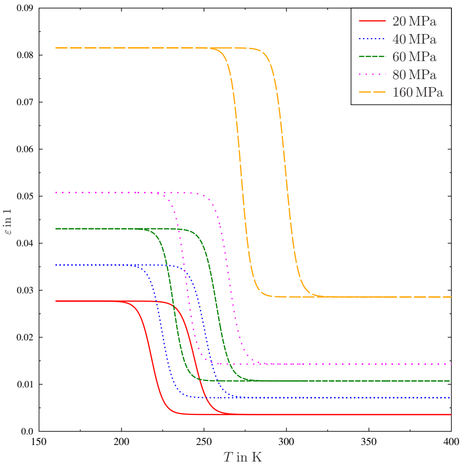

**Fig. 1** Total strain $\varepsilon(T,\sigma)$

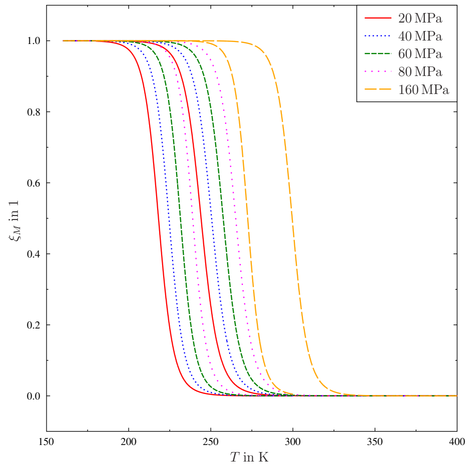

**Fig. 2** Martensitic phase fraction $\xi_M(T,\sigma)$

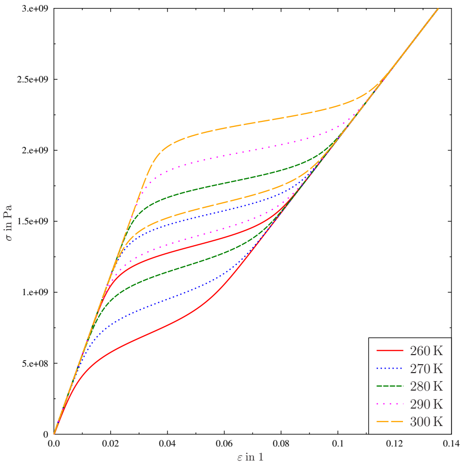

**Fig. 3** Hysteretic stress-strain behaviour, $\sigma(T,\varepsilon)$

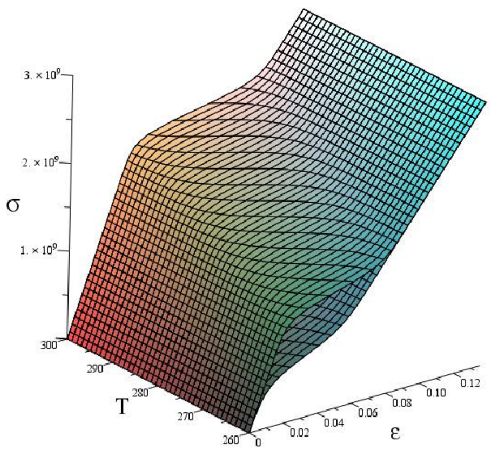

**Fig. 4** Surface of thermomechanical states, $\sigma = f(T,\varepsilon)$

increases. The presented model can predict this behavior (Fig.1). Due to the fact that all results of the model are available in an analytical form, it is straightforward to calculate the enclosed area and further quantities like strain and phase fraction rates [5]. Furthermore this form allows a more computational efficient treatment of vibration, stress wave and damping problems.

## 4 CONCLUSION

In the current study the application of a recently developed method based on statistical physics for the calculation of stress dependent hysteretic behavior of SMA polycrystals is shown. The information of the phase changes at different temperatures for cooling and heating is included in the experimentally obtained heat capacity of SMAs, an artificial phase triggering is not required. The model is capable to determine the hysteretic behavior of SMAs in a correct manner.

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