ADDITIVITY, METALLURGICAL RECOVERY, AND GRADE

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ABSTRACT

Proper estimation of the metallurgical recovery is very important for the assessment of the economical value of a mining business. As this quantity is non-additive, it is not possible to model its spatial variability or to perform its estimation directly. In the present work, we first recall that additivity can concern the intrinsic nature of point-support quantities, and not only the usual support effect encountered in data sets mixing samples with different sizes. Then, on a practical study based on sulphide copper data, we show when non-additive practices have an impact on the results and when they do not. Finally, we open a discussion where we explain how to proceed to assess metallurgical recovery.

INTRODUCTION

Since the early 90's Codelco has been preoccupied making efforts to understand the spatial variability of some relevant metallurgical variables such as recovery, Bond Index, and Starkey Index. The consideration of the spatial variability of those variables has very important economical implications for the mining business (Carrasco and Tapia, 1998; Pease et al., 1998; Caceres et al., 2006).

Some practical problems arise when assessing the local variability of metallurgical variables:

- Generally the samples are not at constant support.
- Sometimes, in order to get enough weight to perform several tests, several increments coming from very different locations are combined in a composite sample, so that the spatial location is lost.
- Metallurgical recovery is not additive, therefore the assessment of its spatial variability is not direct.
- Generally, the available number of samples is not sufficient to do a proper assessment.
- The spatial models of the geological variables which control the metallurgical behaviour such as lithology, alteration, texture, liberation factor, surface properties and ore zones are not available or are not taken into account.

- The scale process to forecast the mill results from the in situ simulated or estimated point values is not well known (Dance et al., 2003).

The main objective of this paper is to give some sound solution to the assessment of the *spatial variability* of metallurgical recovery taking into account some of the previously mentioned constraints.

ADDITIVITY

Quantities are said to be *additive* if the averaged quantity equals the average of the quantities.

Additivity can concern quantities known over some support. See the example of the grade $Z(V)$ over a block V (tonnage T, metal Q) *with a non-constant density*. Given two blocks V_1 and V_2 having the same volume (but different densities), the average of the two grades Q_1/T_1 and Q_2/T_2 equals $(Q_1/T_1 + Q_2/T_2) / 2$ while the averaged quantity over the big block $V_1 \cup V_2$ equals $(Q_1 + Q_2) / (T_1 + T_2)$.

Another pedagogic example is the inverse of grade (even when the density is constant). The average of the two inverses $1 / Z(V_1)$ and $1 / Z(V_2)$ equals $(T/Q_1 + T/Q_2) / 2$ while the averaged inverse of grade over the big block $V_1 \cup V_2$ equals $2T / (Q_1 + Q_2)$. This is different if metal quantities are different.

But additivity can also concern quantities with a *point support*. Instead of the usual permeability example, consider the example of the variable Colour (denoted $C(x)$) in chalk industry. It quantifies the quality of the chalk and is directly associated with economical considerations. It can be categorized (quality 1, 2, …, n) or continuous, lying for example from 0 (black, or the greyest chalk) to 1 (pure, white), with continuous intermediate tints of grey.

Let us now consider the problem of point estimation of $C(x_0)$ given two samples $C(x_1)$ and $C(x_2)$, such that x_0 is the midpoint of $[x_1, x_2]$. Any linear estimate of $C(x_0)$ will give equal weights to $C(x_1)$ and $C(x_2)$. Does it have a sense? The answer is no. If for example $C(x_1) = 1$ and $C(x_2) = 0$, the mixing of both will not give an intermediate grey ($C(x_0) = 0.5$) but a tint of grey much closer to 0 (black). To obtain an intermediate grey ($C(x_0) = 0.5$), we will have to mix a big amount of white with a very small amount of black. Averaging colours should be done according to physical properties which are very complex. In other words, the link between the different values taken by this variable is non-linear, the reason why it is not additive, even for a point support. A spatial average has no sense. Therefore, a kriging neither. We must keep in mind this example when we estimate a variable, or even just consider its variations in space: Does it have a sense?

METALLURGICAL RECOVERY

The recovery of sulphide ore is obtained experimentally in small-scale laboratory tests. Such a test transforms the ore in a concentrate plus a tail, and provides us with the head grade Z_H, concentrate grade Z_C, and tail grade Z_T. The conservation of the ore quantity T and metal quantity Q implies that

$$T_H = T_C + T_T$$
$$Q_H = Q_C + Q_T$$

Since the quantity of metal is the product of the ore tonnage by the grade, if the ore density is constant, which will be assumed here, this is equivalent to

$$T_H = T_C + T_T$$
$$Z_H T_H = Z_C T_C + Z_T T_T$$

Let r denote the proportion of the in situ ore which constitutes the concentrate, also called "weight recovery":

$$r = \frac{T_C}{T_H}$$

The second equation of the above system can be expressed as

$$Z_H = r\, Z_C + (1 - r)\, Z_T$$

which leads to

$$r = \frac{Z_H - Z_T}{Z_C - Z_T}$$

As $Z_T < Z_H < Z_C$, we can interpret this formula as follows: The head grade Z_H divides the interval $[Z_T, Z_C]$ in two intervals $[Z_T, Z_H]$ and $[Z_H, Z_C]$ whose lengths are proportional to the concentrate and tail tonnages, respectively.

Similarly, the recovery ratio R is defined as the ratio of the quantity of metal in the concentrate to the total quantity of metal:

$$R = \frac{Q_C}{Q_H} = \frac{Z_C T_C}{Z_H T_H}$$

Since T_C / T_H is by definition the ratio r, R can be expressed by

$$R = \frac{Z_C}{Z_H} r = \frac{Z_C}{Z_H} \frac{Z_H - Z_T}{Z_C - Z_T}$$

Let us finally define the (in situ) recovered grade Z_R as the product of the recovery by the head grade:

$$Z_R = R\, Z_H = r\, Z_C \tag{1}$$

This is the variable of interest if we want to estimate the recovered metal quantity (the complement $Z_H - Z_R = (1 - R)\, Z_H = (1 - r)\, Z_T$ represents the in situ grade which is lost in the tail).

The recovery is not an additive variable. If two similar blocks have recoveries R_1 and R_2, respectively, the recovery of the superblock composed of these two blocks is not $(R_1 + R_2) / 2$.

Z_H is of course an additive variable but, contrarily to intuition, Z_C and Z_T are not. More precisely, they would be additive variables if we would consider samples taken in the concentrate or in the tail, but here the coordinates associated to our data are in situ locations, and the concentrate weight and tail weight associated to a core sample are smaller than the core weight and vary from one core to the other. The corresponding additive variables are in fact the recovered grade Z_R and its complement $Z_H - Z_R$.

Conclusion: The set of three basic variables (Z_H, Z_C, Z_T), where Z_H is the sole additive variable, shall be replaced by a set of three additive variables, namely Z_H, r, and Z_R.

EXPERIMENTATIONS

Presentation of the Data

There are 1112 samples covering 1300 m along X, 3900 m along Y and 1400 m in depth. Data are split in three mineral zones (206, 207, and 409) and main tests have been applied to the last one represented by 671 samples (Fig. 1).

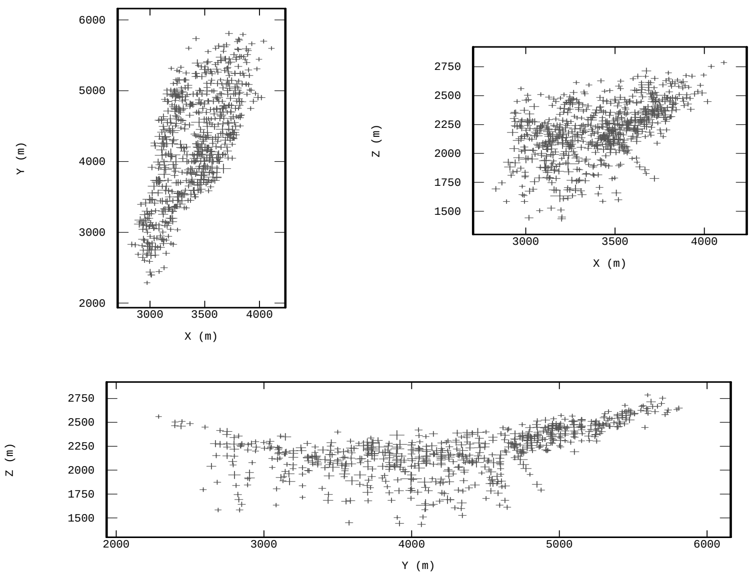


Figure 1: 671 samples of mineral zone 409 (projection on the orthogonal planes XY, XZ, and YZ).

Each sample contains the Recovery R, the Head grade Z_H, Concentrate grade Z_C, and Tail grade Z_T. According to the previous formulas, we calculate and study the ratio r (small letter, called here r_ratio to prevent from any confusion with Recovery R) and the recovered grade Z_R.

The scatter diagrams between Z_H, Z_R and r_ratio (Fig. 2) show a strong correlation of 0.995 between Z_H and Z_R with a very low dispersion of their difference. We obviously have $Z_R < Z_H$. The correlation with *r*_ratio is not very large (about 0.4). The high correlation between Z_H and Z_R implies:

- In this particular case the recovery process has been very efficient. The slope of the regression between Z_R and Z_H is the mean metallurgical recovery of the unit.
- The geological controls for the copper mineralization are as well controls for the recovered grade Z_R.
- As a consequence the estimation and simulation domains for both variables should be the same.

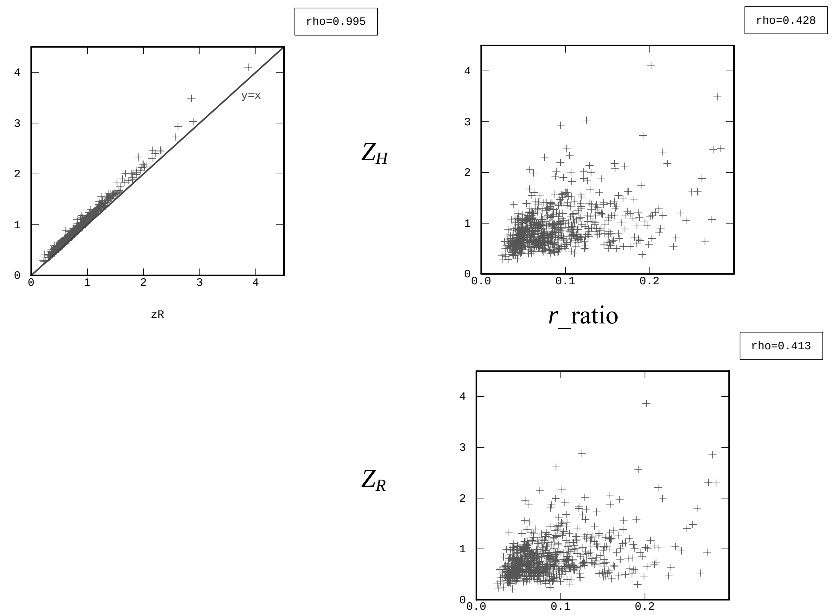
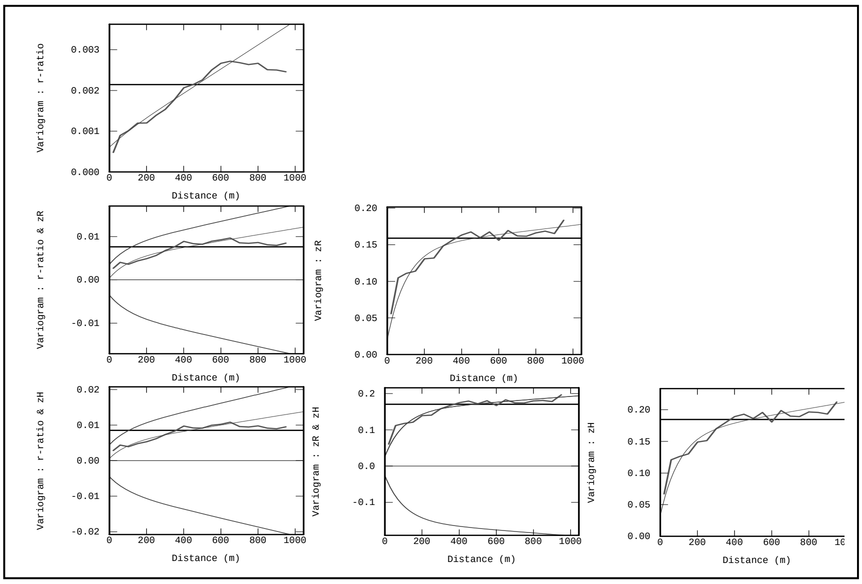


Figure 2: Scatter diagrams between Z_H and Z_R (top left), Z_H and *r*_ratio (top right), Z_R and *r*_ratio (bottom).

While the experimental variograms of Z_H and Z_R are isotropic and stationary (Fig. 3), that of *r*_ratio has a linear tendency, with an important nugget effect (about 25 % of the experimental variance). The cross-variograms confirm the previous remarks: the behaviour between the cross-variograms of Z_H and Z_R is similar to their direct variograms, whereas the cross-variograms between *r*_ratio and the other two variables are a mixture of the behaviours seen in the direct variograms, with a medium correlation.



γ_r $\gamma_{r,ZR}$, $\gamma_{r,ZH}$ γ_{ZR} $\gamma_{ZR,ZH}$ γ_{ZH}

Figure 3: Experimental variograms of *r*_ratio (top), Z_R (middle right) and Z_H (bottom right). Cross-variograms complete the table. Fitted curves represent tendencies, not models.

Experiment 1: Global Recovery

If we want to calculate the recovery *R* at the scale of the deposit, we can imagine to use point values for this, considering that the associated sampling constitutes an acceptable discretization of the deposit. By averaging the different point recoveries known at samples, we obtain a global estimation R^* equal to 88.8% while the right calculation, which consists in dividing the average of Z_R by the average of Z_H (supposing that these averages are estimated without error), gives 89.3%. The difference cannot be neglected at the scale of the deposit. We have done the calculation for the other mineral zones and see important differences between both calculations in all cases (Table 1).

Table 1: Global recovery for three mineral zones. R^* is the average of point recovery, *R* the average of Z_R divided by the average of Z_H

Mineral zone	Number of samples	Variance of Z_H	Mean of Z_H	Variation coefficient	R^* Average of recoveries (biased)	R Ratio of averages (correct)
409	671	0.18	0.91	47%	88.8%	89.3%
207	394	0.32	1.16	49%	88.8%	89.8%
206	47	1.21	2.22	49%	86.3%	86.9%

Experiment 2: Local Recovery

Now we focus on a restricted 200×200×50 m^3 area (Fig. 4).

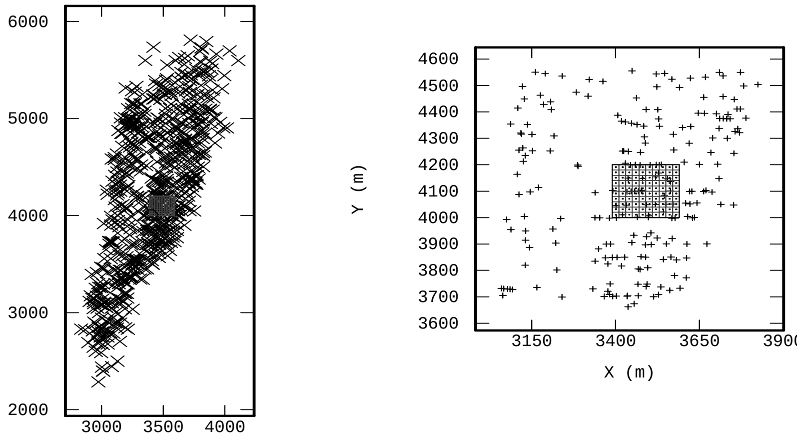


Figure 4: In the centre (3400 < X < 3600, 4000 < Y < 4200) the restricted area where point recoveries have been simulated. This area is divided in 500 blocks. Each block is discretized in 500 nodes. On each node, 20 cosimulations values of Z_H, Z_R, and r_ratio are calculated.

This area has been divided in 500 blocks V_j, each of which is discretized in 500 nodes. For each node, 20 conditional cosimulations of Z_R, Z_H and r_ratio, are done, which reproduce the general histograms and variograms deduced from the initial data. Each simulation allows calculating the block recovery in two ways:

- Additive procedure: For each block V_i, $Z_R(V_i)$ and $Z_H(V_i)$ are calculated by averaging the point values contained in V_i. Then we obtain the recovery of the block, denoted $R(V_i)$, as the ratio of these quantities.
- Non-additive procedure: For each node, point recoveries are calculated, using formula (1). The recovery of the block V_i is obtained by averaging the 500 point recoveries contained in V_i.

Again, the exercise consists in comparing the ratio of two averages to the average of a ratio. Figure 5 shows the results for three realizations. Each time, the correlation between the non-additive estimator and the correct one is equal to 1, the distributions are the same, and differences on the bounds of the values and the average are of the order of the third decimal. These differences are not significant.

Explanations

So we have here two simple experiments which give different results concerning the impact of additivity. The second one, the evaluation of the influence of additivity at the scale of a block, consists in the comparison, for each block V, of an average of ratios

$$\left(\frac{Z_R}{Z_H}\right)(V) = \frac{1}{n}\sum_j \frac{Z_R(x_j)}{Z_H(x_j)}$$

and a ratio of averages

$$\frac{Z_R(V)}{Z_H(V)} = \frac{\frac{1}{n}\sum_j Z_R(x_j)}{\frac{1}{n}\sum_j Z_H(x_j)}$$

where n is the number of discretization points x_j of V.

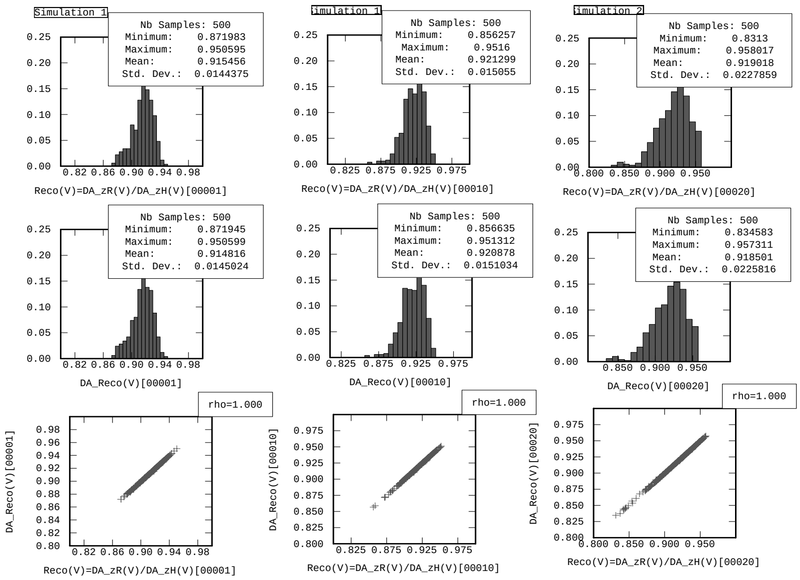


Figure 5: Cosimulations n° 1 (left column), 10 (middle) and 20 (right column). Upper histograms concern the distribution of block recoveries $R(V)$ obtained by the additive calculation, middle ones the non-additive procedure. Lower figures are scatter diagrams between the correct calculation (horizontal axis) and the biased one (vertical axis). No significant difference can be detected. The impact of non-additivity can be neglected.

There are two situations where both formulas are equivalent: i) the head grade $Z_H(x_j)$ is constant; ii) the ratio $Z_R(x_j) / Z_H(x_j)$ is constant. Let us examine if we are close to these situations, for a block support and at a global scale, respectively.

1: Point-to-block dispersion of Z_H

The variability of point values within a block V is measured by the point-to-block dispersion variance $\sigma^2(0 \mid V)$, which is equal to the average variogram between two points uniformly and independently distributed in block V:

$$\sigma^2(0\,|\,V) = \overline{\gamma}(V,V)$$

Since the size of V is 20×20×10 m^3, the distance between two points in V lies between 0 and 30 m. The variogram of head grades (Fig. 3, bottom right) shows that its average value for such distances is of the order of 0.05. Then, even if the local distribution of point values of Z_H within V may vary significantly from a block to another one, within each block V, most point values belong to the narrow interval $[Z_H(V) - 0.45\ ,\ Z_H(V) + 0.45]$. To get an order of magnitude, this gives a variation coefficient equal to 17% when we refer to the average of point values of Z_H all over the domain (which is a crude approximation).

Now if we increase the size of the block, the point-to-block dispersion variance increases until reaching the experimental variance of the data, so that Z_H is far from being constant when we consider the global scale. This is why additivity has an impact in the case of global recovery calculation.

2: Conditional variance of Z_R given Z_H

The strong correlation coefficient between Z_R and Z_H (0.995) indicates a general behaviour of Z_R following Z_H linearly and also that 99% of the variance of Z_R is explained by its linear dependency on Z_H. The cross-variogram of Z_R and Z_H (Fig. 3, bottom middle) shows that their spatial correlation is very high, even for the nugget effect component. But the most important fact that can be observed on the scatter diagram of Z_R and Z_H (Fig. 2, top left) is that the conditional variance of Z_R, given Z_H is small for any Z_H, even if it tends to increase with the head grades. As a consequence the ratio $Z_R(x_j)/Z_H(x_j)$ varies only slightly with $Z_H(x_j)$, and is approximately constant when Z_H does not vary too much, and in particular at the scale of a block V. This is the reason why, at such a small scale, we see little difference between both calculations. At a global scale, the ratio $Z_R(x)/Z_H(x)$ displays some variations so that additivity has a significant impact.

CONCLUSIONS

These two simple experiments, which consist in comparing ratio of averages to average of ratios, show that additivity can or not have an impact on the result, depending on the variability of the quantities involved in the ratios, and the scale of the calculation. In the case of global recovery, the variance involved equals 3 times the variance involved in the case of local recovery, and this sole difference causes the emergency of the influence of additivity. This result must incite us, one time for all, to give up illicit calculation as their validity may vary without control.

Consequently, we have to face the problem of recovery estimation, and its use in the mineral industry. In practice, there are many samples where head grades Z_H are known, but very few laboratory tests where Z_R, r_ratio and R can be calculated (around 1 laboratory test for 100 exploratory measurements). On the one hand, geologists do a kriging of head grades to build the block model. On the other hand, process engineers do a kriging (or average) of recoveries using the few laboratory tests, and the recovery grade estimator is obtained by multiplying both estimators. This is certainly the worse procedure we can imagine, as this product first uses an illicit estimation of *R*, and secondly it is not optimal and may be biased.

The correct procedure must use cosimulations of additive variables Z_H, Z_R and *r*_ratio. Then, if we are interested by block recoveries, we must average Z_H and Z_R over each block, and then divide; and if we are interested by the point recovery all over the domain, we must calculate its conditional expectation estimator. We build something like 100 cosimulations, calculate point recovery for each simulation (at each node), and then average the different realizations of *R* at each node: this time, this is a statistical average, not a spatial one, and it is licit.

Finally, all our work here did consist in replacing the problematic triplet head grade, concentrate and tail grade by the benefit triplet head grade, recovery grade and weight recovery.

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