Controlling the ultimate openings residues for a robust delineation of fragmented rocks

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CONTROLLING THE ULTIMATE OPENING RESIDUES FOR A ROBUST DELINEATION OF FRAGMENTED ROCKS

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ABSTRACT

These last years image analysis has started to be thoroughly used for size distribution quality control in mineral industry. Compared to sifting, it has the advantage of reducing the interference with production, of measuring automatically and on-line, and finally of improving the representativeness. Nevertheless, measurement is confronted to various problems, among which the correct extraction of fragments projected areas (2D), more particularly related to problems of over-segmentation and fusion. Indeed, images taken under natural conditions of lighting and positioning of fragments are very complex and noisy to be properly processed. Usual filtering methods, based on linear and non-linear filters, are not appropriate for this kind of analysis. In this paper, a new filtering method is presented. It is based on a new topographic interpretation of image structures (relative grey levels), and uses residual morphological transformations (ultimate opening residues and its associated function). A control of the ultimate opening construction is carried out with the help of a geometrical criterion applied on fragments shapes. Thus, it is possible to calculate a new and dramatically filtered associated function, giving access to relevant markers for the fragments. Thanks to the evaluation of the method on images affected by various types of noise (fine particles, strong texturation, etc), its robustness and efficiency can be proved. Lastly, the faithful delineation of fragments, carried out thanks to relevant marking, leads to the extraction of correct 2D information, totally usable for size attribution (1D) and volume reconstruction (3D).

Keywords: Delineation of fragmented rocks, Residual transformations, Ultimate opening, Granulometric function, Maximum inscribed discs, Fine particles segmentation, Morphological marking.

INTRODUCTION

The initial images are acquired by a digital B&W camera (256 grey levels). Sizes of the fragments present on the images vary according to the place of acquisition (cut-down heap, conveying truck, storage hopper and belt conveyor). In the same way, and for the same reason, the quantity of the very small sizes fragments (fines), subject to segregation and forming clusters at bottom of image or above large fragments, varies from one image to another. Generally, fragments are separated by shaded zones. These separations can actually be either empty zones or zones of fines hidden by the shadows of big fragments. It is the contrast generated by the difference in grey levels between these zones and the average grey level of the fragments surfaces which defines the fragments contours. However, an efficient rock contour extraction must address the difficult problem of the removal of non-relevant features (noise) in the images. Indeed, rock fragments images are rather complex and it is unlikely that basic operators as thresholding would be sufficiently efficient to extract these fragments. The following analysis summarizes the main factors of noise. The acquisition of the images under natural illumination leads to luminosity inhomogeneities in the field of analysis. Surfaces of fragments have not a constant average grey level. They are characterized by small variations of grey levels of and by a clear and dark texturation, more or less important according to the rock type. This texture produces many local extrema. The presence of fine particles on bigger fragments surfaces interferes with the extraction of relevant fragments markers. Many filtering methods were tested to smooth images of rocks fragments and to prepare them for a correct extraction of contours. Hereunder, we will describe the state of the art of the most known rocks fragments image processing software in the literature, namely: FragScan (Schleifer J *et al.*, 1996), Split (Girdner *et al.*, 1996) and WipFrag (Palangio, 1985). FragScan, developed at Paris School of mines, was conceived initially to estimate the size distribution of the whole heap just after the blasting. The 2D treatments consist of a pre-filtering by a local homogenization followed by an automatic thresholding using the maximization of the variance between classes. The resulting images being binary images, the recognition of fragments contours was only partial, leading to not correct sizes measurements. Split was developed at the

University of Arizona in the beginning of the nineties. The treatments aim at automatically delineate rock fragments from images acquired on the belt conveyor. Due to the noise, the 2D analysis produces most of the time over-segmented fragments (case of large fragments) or, on the contrary, merged regions (fines). In spite of the use of a geometrical criterion based on the distance function, these over-segmentation and merging problems persist. In order to get rid of them a manual delineation of the fragments is needed. However this largely harms the automation of measurement. WipFrag, was developed at the Waterloo University in Canada. 2D treatments initially try to partially detect contours then to close them by vicinity-based techniques. Hand corrections are also made to manage errors of delineation. The approach presented in this work makes use of a different procedure. It starts with rocks fragments images simplification, by extracting for each fragment the maximum disc inscribed in its interior. To achieve this, we control the construction of the granulometric function associated to the ultimate opening of the image. As this building preserves the main topographic characteristics of the relevant features in the image, dark and white noisy textures are eliminated without deteriorating fragments contours, thus avoiding fusion and over-segmentation problems. Based on a residual morphological transformation, i.e. the ultimate opening by discs and its associated function (or granulometric function) (Beucher, 2007), this approach allows a local comparison and an extraction of the greatest relative grey levels of the various structures as well as the preservation of the largest structures corresponding to the fragments.

MATERIAL AND METHOD

RESIDUAL TRANSFORMATIONS, ULTIMATE OPENINGS

Given two families of transformations (the primitives) depending on a parameter i ($i \in N = [1,n]$), ψ_i and ξ_i, with $\psi_i \geq \xi_i$, we define the residue of size i between these two primitives at every point x of the image as the difference

$$\rho_i(x) = \psi_i(x) - \xi_i(x) \quad (1)$$

A residual transformation θ is defined as:

$$\theta = \underset{i \in N}{Sup}(\psi_i - \xi_i) \quad (2)$$

This transformation computes locally, at each point of the image, the greatest residue of the two sequences of primitive transforms. A function Φ, called associated function is linked to this transformation. The value of Φ at point x is equal to the index i for which $\rho_i(x)$ is strictly positive and maximal.

$$\Phi = argmax(\rho_i) = argmax(\psi_i - \xi_i) \quad (3)$$

$$\{\Phi(x) = i : \rho_i(x) > 0 \text{ and maximal}\} \quad (4)$$

If this maximum appears for several values of i, only the highest value will be retained:

$$\Phi(x) = \{max(i) : \rho_i(x) > 0 \text{ and maximal}\} \quad (5)$$

Let us consider the residual operator υ where the primitives ψ_i and ξ_i are respectively an opening by balls of size $i-1$ and an opening by balls of size i:

$$\begin{cases} \psi_i & = & \gamma_{i-1} \\ \xi_i & = & \gamma_i \end{cases} \quad (6)$$

$$\upsilon = \underset{i \in N}{Sup}(\gamma_{i-1} - \gamma_i) \quad (7)$$

Where $N = [1, n+1]$, n being the size of the maximal non-empty opening. We have, by definition, $\upsilon_0 = I$, identity function. This operator replaces the initial greytone image by a union of the most significant cylinders included in the sub-graph of the initial function. The associated function ς, called granulometric function, presents also a big interest. At every point x, ς is equal to the radius of the most significant cylinder covering x.

The ultimate opening performs a local comparison of the relative grey levels of the image structures. The computation of the ultimate opening on a noisy image of rock fragments replaces each fragment by a stacking of disks (Fig. 1-b). These disks are maximal. Their sizes correspond to the largest cylinders, with heights equal to the relative grey levels of image structures, included in the image. We can notice that, compared with the original image, (Fig. 1-a), fragments surfaces are more homogeneous in the ultimate opening image.

Building the ultimate opening leads to a dramatic simplification of the initial image. However, despite this important smoothing, fragments surfaces remain affected by residual noises that appear as white peaks on fragments surfaces in the ultimate opening. On the granulometric function (Fig. 1-c), the same peaks appear as holes. The occurrence of holes is due to some structures (black and white textures on rocks and fine particles on large fragments) which present high relative grey levels. Therefore, these features are not filtered and remain in the ultimate opening.

The main interest of the granulometric function associated to the ultimate opening transform lies in the possibility to easily extract the maximum discs inscribed in each rock fragment. A simple procedure able to achieve this would be to perform an opening of size $i-1$ on the corresponding threshold i of the granulometric function. However, the residual noise mentioned above, by introducing holes and artifacts in the different layers of the granulometric function, compromises this simple approach because the size of the elements which correspond to a value i in the granulometric function, is not necessarily equal to $i-1$. Moreover, even if the holes are filled, many other defects like open porosity remain, which, in the end, leads to the fact that the sizes of many rock fragments do not correspond to the values of the granulometric function.

In order to cope with this problem, a solution consists in controlling the construction of the granulometric function in order to ensure that the elements likely to correspond to fragments are not holed and are of right sizes. This selection leads to the building of a new granulometric function where the maximal discs inscribed in each rock fragment can be extracted, as proposed above, by openings γ_{i-1} applied to each threshold i of this granulometric function (Outal, 2006).

CONTROLLING THE CONSTRUCTION OF THE ULTIMATE OPENING

CONSTRUCTING THE ULTIMATE OPENING

Let us come back to the construction of the ultimate opening and of the granulometric function. This construction is made layer by layer. Let us denote $\upsilon_{k-1}(f)$ the ultimate opening after $(k-1)$ building steps:

$$\upsilon_{k-1}(f) = \underset{i\in[1,k-1]}{Sup}\left(\gamma_{i-1}(f) - \gamma_i(f)\right) \tag{8}$$

The corresponding temporary granulometric function is denoted ς_k. At step k, the ultimate opening becomes $\upsilon_k(f)$:

$$\begin{aligned} \upsilon_k(f) &= \underset{i\in[1,k]}{Sup}\left(\gamma_{i-1}(f) - \gamma_i(f)\right) \\ &= Sup[(\gamma_{k-1}(f) - \gamma_k(f)), \upsilon_{k-1}(f)] \end{aligned} \tag{9}$$

From $k-1$ to k, a new layer denoted χ_k has been added (although it is not a simple arithmetic addition) to the ultimate opening. χ_k is not simply the residue ρ_k but only a part of it. χ_k is the part of ρ_k whose support M_k is equal to the threshold k of the granulometric function ς_k.

$$M_k = \{x : \varsigma_k(x) = k\} \tag{10}$$

This set corresponds to the pixels x of the image for which $\upsilon_k(f(x)) \geq \upsilon_{k-1}(f(x))$. If m_k is the corresponding numerical (or masking) function defined as:

$$\begin{cases} m_k(x) &= 255, \text{ if } x \in M_k \\ &= 0, \text{ if not} \end{cases}$$

(The maximum possible grey level in the image is assumed to be equal to 255). Then, χ_k is defined as:

$$\chi_k = Inf(\gamma_{k-1} - \gamma_k, m_k) \tag{11}$$

It corresponds to the part of the residue ρ_k included in the mask M_k. The ultimate opening is then defined as:

$$\upsilon = \underset{k\in N}{Sup}(\chi_k) \tag{12}$$

It can also be defined iteratively as:

$$\begin{cases} \upsilon_0 &= 0 \\ \upsilon_k &= Sup[Inf(\upsilon_{k-1}, 255 - m_k), \chi_k] \end{cases} \tag{13}$$

υ_k is build at step k by keeping first the parts of υ_{k-1} which are not included in M_k, then by adding the new component χ_k. This latter formula is quite complicated and useless when computing the standard ultimate opening because in M_k^c, χ_k is always lower than υ_{k-1}. However, in the sequel, this definition will be needful to build next transformations. In the same way, the granulometric function ς is build iteratively step by step:

$$\begin{cases} \varsigma_0 &= 0 \\ \varsigma_k &= Sup[Inf(k, m_k), \varsigma_{k-1}] \end{cases} \tag{14}$$

Starting from the previous building step ς_{k-1}, the mask M_k is given the value k before being incorporated to ς_{k-1}.

THE CONTROL TRANSFORMATION Ψ

The main interest of these two last definitions comes from the possibility to iteratively build the ultimate opening and the granulometric function from the sets M_k. Therefore, if we modify these sets, we can build new ultimate openings and new granulometric functions. Now, the procedure, which aims at extracting the maximal disks inside rocks fragments, consists precisely in selecting among the initial sets M_k those that, firstly, can be used as markers of fragments. Secondly, these sets will be modified and used to build a new ultimate opening and a new granulometric function. Let us denote Ψ the transformation that, starting from an initial set M_k produces a new set M_k'. Then, the overall process can be summarized as follows: For each size k, from 1 to $n+1$, n being the maximum size of the fragments in the image:

- Define the mask : M_k;

- Compute the modification of the mask :

$$M_k^{'} = \Psi(M_k) \tag{15}$$

- Define the new granulometric function $\varsigma_k^{'}$:

$$\begin{cases} \varsigma_0^{'} & = & 0 \\ \varsigma_k^{'} & = & Sup[Inf(k, m_k^{'}), \varsigma_{k-1}^{'}] \\ & = & \underset{i \in [1,k]}{Sup}\,[Inf(i, m_i^{'})] \end{cases} \tag{16}$$

with

$$\begin{cases} m_k^{'}(x) & = & 255, \text{ if } x \in M_k^{'} \\ & = & 0, \text{ if not} \end{cases} \tag{17}$$

- Define the new ultimate opening $\upsilon_k^{'}$:

$$\upsilon_k^{'} = Sup[Inf(\upsilon_{k-1}^{'}, 255 - m_k^{'}), \chi_k^{'}] \tag{18}$$

$$\text{with : } \chi_k^{'} = Inf(\gamma_{k-1} - \gamma_k, m_k^{'})$$

Note that the construction of the new ultimate opening is not necessary since the maximal disks embedded in the fragments are extracted from the granulometric function. They simply correspond to the maxima of the function $\varsigma^{'}$.

Let us describe in details the successive steps of the transformation Ψ. As already mentioned before, the binary sets M_k are often polluted by holes. Therefore, the first step Ψ_1 of the transformation Ψ consists in closing the holes inside M_k. Let us denote η this operation (Fig. 1-e):

$$\Psi_1(M_k) = \eta(M_k) \tag{19}$$

As shown in a previous example, despite the closing of holes, the size of the new set is often lower than $(k-1)$. This is due to the fact that an open porosity remains after hole closing. However, the sufficiently large connected components can be considered as suitable markers of some maximal disks in spite of their damaged shape. So, the second step Ψ_2 of the Ψ transform consists in applying a size criterion to keep parts of the set M_k that may be candidates as maximal disks markers. This size criterion is an opening of size $(\frac{k}{a} + b)$ (Fig. 1-f):

$$\Psi_2 = \gamma_{(\frac{k}{a}+b)} \tag{20}$$

This operation is the only one in the entire Ψ transform that is parametric. The parameters a and b are two constants which depend on the petrographic characteristics of the fragmented rocks. They are fixed once and for all the samples belonging to the same quarry face. The value a indicates the percentage of size reduction allowed and b gives the minimum size of the set under which it is not taken into account. The markers obtained after this second step are then used to rebuild the maximal disks of size $k-1$. These disks are contained in the support R_k of the residue $\gamma_{k-1}(f) - \gamma_k(f)$ (Fig. 1-h):

$$R_k = \{x : \gamma_{k-1}(f) - \gamma_k(f) > 0\} \tag{21}$$

More precisely, the disks which size is at least equal to $k-1$ and contained in R_k can be isolated by an opening of size $k-1$ of R_k (Fig. 1-i).

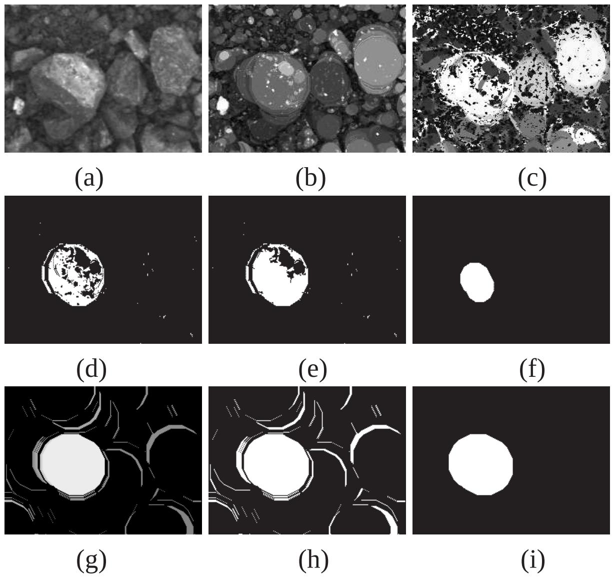

Fig. 1. ***Controlling the ultimate opening (*Ψ_1 *and* Ψ_2*)***.

For extracting among the disks those which are marked by the marker sets previously obtained, a reconstruction approach based upon the use of critical maximal disks will be applied (approaches based on geodesic dilations of the marker sets are not relevant). The notion of critical disk was introduced in (Beucher *et al.*, 1990). A maximal disk is critical when it cannot be covered with any combination of the other maximal disks. In $\mathbb{R}^n$, one can show easily that, if we consider Euclidean disks (balls), the set of critical disks of X is unique. However, if the disks are polyhedra, this uniqueness is not verified. Therefore, in digital spaces, a restricted definition of a critical disk B_i is used and is the following:

$$B_i \text{ critical } : \nexists\, J = \{j_1, ..., j_N : j_k \neq i\} / B_i \subset \bigcup_{j \in J} B_j \tag{22}$$

The granulometric function of the set X can be used to filter the critical disks. More details on this filtering can be found in (Outal, 2006; Beucher, 2007).

The third step Ψ_3 of the Ψ transform will then consist in keeping among the critical disks of each connected component of $\gamma_{k-1}(R_k)$ the disk in which the marker set obtained after step Ψ_2 fits best. To achieve this step, successive erosions of the marker set are performed and critical disks centers (Fig. 2-d) falling out of these erosions are removed until only one center remains. In other words, only the critical disk center closest to the last erosion of the marker set is kept (Fig. 2-e). We can write this third step as:

$$\Psi_3 = Rec_{cd}(M; \gamma_{k-1}(R_k)) \tag{23}$$

Where M are the filtered marker sets obtained after the transforms Ψ_1 and Ψ_2. Thus, the transformation Ψ which modifies the original marker set M_k and produces the new markers $M_k^{'}$ can be written as follows:

$$M_k^{'} = \Psi(M_k) = \Psi 3 \circ \Psi_2 \circ \Psi_1(M_k) \tag{24}$$

$$M_k^{'} = \Psi(M_k) = Rec_{cd}[(\gamma_{(\frac{k}{a}+b)}(\eta(M_k); \gamma_{k-1}(R_k))] \tag{25}$$

The transformation Ψ is obviously rather complex. However, it can be achieved only by means of the initial granulometric function and the whole transform is controlled by a unique parametric setting, the doublet (a, b) which, itself, depends only on the type of rocks under study. At the end, a new granulometric function $\varsigma^{'}$ can be built from the new sequence of markers $M_k^{'}$ (Fig. 2-f), according to the formule 16.

This granulometric function corresponds to the stacking of the labelled relevant disks calculated for each size k (Fig. 2-h). The maximum disks inscribed inside each rock fragment will thus correspond to the regional maxima of the granulometric function (Fig. 2-i).

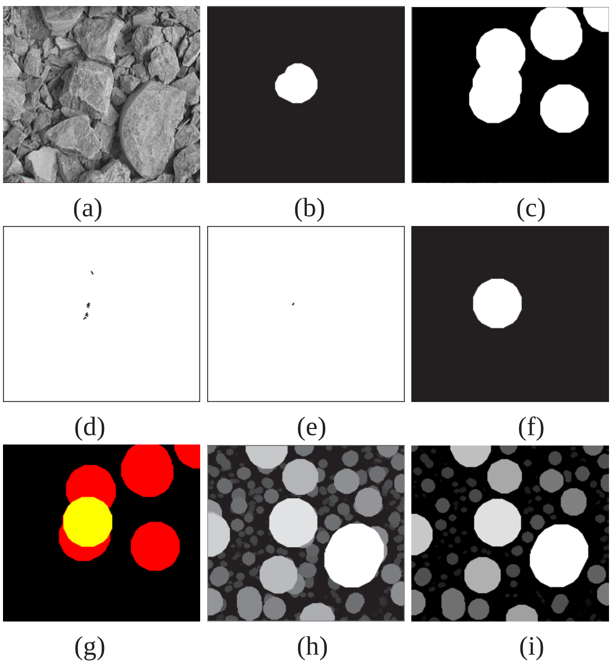

Fig. 2. Ψ_3 ***and the new granulometric function*** $\varsigma^{'}$.

RESULTS AND DISCUSSION

This process has been applied, for validation, to various images representing the most frequent problematic cases encountered in rocks fragments image processing. In particular, the presence of grouped fine particles, non-uniform contrasts in the image and finally the case of images taken under natural lighting have been addressed.

EXTRACTION OF MAXIMUM INSCRIBED DISCS AND FRAGMENTS MARKING

As it has been already mentioned, the major interest of the maximum disks computation lies in their use as markers. These markers could be used for various applications such as counting the visible fragments on the surface or for segmentation purposes. Moreover, as the new granulometric function also provides the sizes of these maximal disks, the markers can be better placed above the fragments by eroding the maximum disks proportionally to their size (half the size of the maximum disk for instance) (Fig. 3-b,e,h).

- The first picture (Fig. 3-a) illustrate the case where a large proportion of fine particles are present in the samples. We can notice that the great problem of detecting fine particles have been resolved. Indeed, practically all the fines are pointed by correct markers (Fig. 3-b).

- The second example (Fig. 3-d) shows a heap of rocks acquired under natural lighting with large variations of contrast throughout the image. Both large fragments and smaller ones are correctly marked. The same result is obtained for fine particles regions, even in natural lightning conditions. In addition, thanks to the applied reconstruction method, based on critical discs, the markers are correctly placed at the center of the fragments projected areas. This is a valuable result when delineating the fragments by a segmentation based upon a controlled watershed.

- The third case image (Fig. 3-g) comes from the MiCa laboratory at University of Liège. It has been realised with the collaboration of the Spin-Off DEIOS (Outal & Pirard, 2006). It represents an image acquired by means of a laser triangulation technique. Although the image aspect is quite different with shadings instead of textures, the process provides a rather good result. Few artifacts due to the presence of more than one marker by fragment (Fig.3-h) can be suppressed by adapting the size criterion, that is the parameters a and b, to the sizes of the fragments present in each image.

SEGMENTATION

The preceding markers can obviously be used to segment the visible fragments in the image by means of a marker-controlled watershed of the gradient image (Fig. 3-c,f,i). Note that, in order to get a better segmentation, some markers corresponding to the image background must be added to the previous marker set. The background may appear if the blocks do not entirely cover the field of analysis. Nevertheless, the background (corresponding to the zero grey level) does not interfere during the computation of the granulometric parameters, namely sizes and projected areas of the fragments, which are determined on the segmented binary image. In addition, for the three analysed samples, the delineation of contours is practically identical to a reference segmentation (manual segmentation). Morerover, the delineation of fine particle regions, thanks to an appropriate marking procedure, is really efficient.

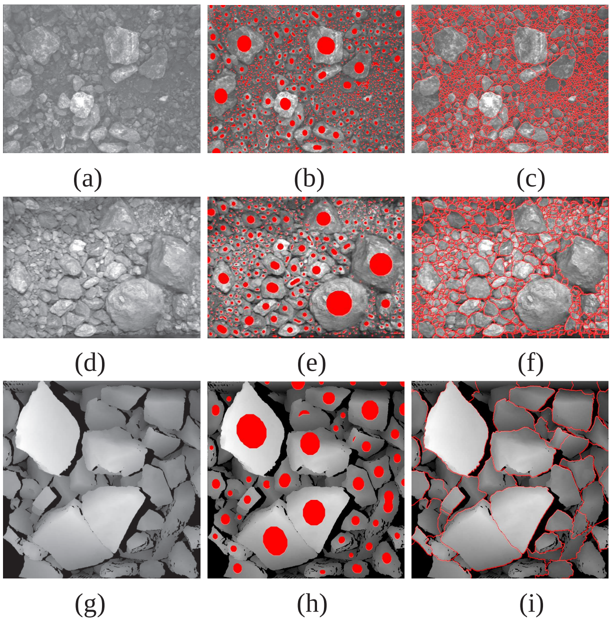

Fig. 3. ***Maximum inscribed discs and delineation***.

CONCLUSION

A new morphological method for filtering the extremely noisy images of fragmented rocks has been developed using the control of the construction of the ultimate openning and its associated function. The method presents the advantage of correctly filter entire regions of images, allowing the correct extraction of the maximum inscribed disk for each fragment visible on the image. On the one hand, the control of the construction of the ultimate openning avoids any loss of useful information and the deterioration of fragments contours during the filtering. On the other hand, the use of inscribed disks as markers of watershed transform leads to a relevant delineation of all the contours. The good results of delineation obtained and validated on various samples show the robustness of the method. In the same way, a suitable extraction of the fine particles regions was achieved. In addition, since the filtering as well as the delineation are automatic, this algorithm can be perfectly inserted in on-line quality controls of fragmentation processes. Moreover, the quality of the projected areas allows a better computation of the real 3D size distributions. These developments involving projected areas and maximum inscribed disks confirm the relevance of this information for a better assignation of sizes (1D) and a better reconstruction of volumes (3D) (Outal *et al.*, 2008).

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