An application of a master-slave algorithm for solving 3D contact problems between deformable bodies in forming processes

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ABSTRACT. *We consider a finite element approximation of frictional contact problem between deformable bodies undergoing large deformations. The fully 3D mechanical coupling problem is expressed with a mixed velocity-pressure formulation. The multi-bodies contact problem is set as a linear complementary problem solved by a penalty method. The corresponding non-penetration condition is approximated using a finite element meshes which do not necessarily fit on the contact zone. The local approach used to take into account unilateral contact on non-matching meshes is an extension of the master-slave algorithm. The mechanical system is solved using iterative methods. The associated model and algorithm are implemented inside the 3D software Forge3®. The selected application is the process of viscoplastic metal forging.*

RÉSUMÉ. *On considère une approximation éléments finis du problème de contact avec frottement entre deux corps déformables en grandes déformations. Le modèle de couplage mécanique fort en 3D est décrit avec une formulation mixte vitesse pression. Le contact entre corps déformables est posé comme un problème complémentaire résolu par une méthode de pénalisation. La condition de non pénétration correspondante est appliquée, sur des maillages éléments finis non coincidents, au niveau local par un algorithme maître esclave. Le système mécanique est résolu par une méthode itérative. Le modèle et l'algorithme associé sont implémentés dans le logiciel Forge3®. L'application présentée est le procédé de forgeage de métaux de rhéologie viscoplastique*

KEYWORDS: *contact, friction, master-slave algorithm, finite element, deformable bodies, penalty formulation.*

MOTS-CLÉS : *contact, frottement, algorithme maître esclave, éléments finis, corps déformables, formulation pénalisée*

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1. Introduction

In continuum mechanics, the contact problems between deformable bodies occur in many applications. Here, we are interested in metal forming process. Numerical simulations of forming processes require a correct numerical interpretation of the finite kinematics involved in the mechanical problem.

Numerical methods for discrete or discretized contact problems, as classically done in constrained problems, are usually divided into two main families: the first one is based on penalty methods, the other one on the duality problem associated to the non-penetration condition. In the bibliography below, we try to present the advantages and the drawbacks of each of them in a general contact case. A better presentation of the state of art in contact problem is synthesized in [CHA 98, WRI 95].

In penalty method [CES 93, PAV 96], the higher the chosen penalty coefficient, the less the interpenetration between the different bodies in contact. On one hand, large penalty coefficient lead to ill-conditionned stiffness matrix and then to bad convergence. On the other hand, small value lead to poor precision. From the numerical standpoint, this method has the advantage to avoid introducing additional variables and therefore leads to solve a system which size is the same as the problem without contact.

A straightforward way to handle the non-penetration condition [FIS 95, PAN 97] consists in checking whether the contact condition is satisfied or not according to the sign of Lagrange multipliers. The Lagrange multipliers have the physical meaning of contact forces. They become additional variables and increase the size of the system. But, theoretically, this contact treatment is exact.

Mixed approaches based on methods such as augmented Lagrange multipliers integrate the two previous methods [ZAV 95, SIM 92]. It amounts to "penalize the Lagrange multipliers". In this case, the penalty coefficient can be small.

In finite element procedures solving contact problems between deformable bodies, each body is often discretized independently. So, the finite element meshes do not coincide on the contact surface. When the meshes do not coincide, the difficulty is to properly impose the contact conditions without over-constraining the numerical problem [FOU 99].

For this purpose, the approximation using the master-slave technique is considered here. One of the antagonistic body is assumed to be the slave body, the other one being the master. The basic idea of this approach was introduced by [BAT 85, HAL 85] and is mainly developed in structural mechanics. Our goal is to extend this model to 3D metal forming simulation. In such application, one body, the part, undergoes large deformations.

The paper is organized as follows. In section 2, we introduce the model describing the mechanical problem for viscoplastic bodies. The mechanical problem is presented in a non-linear kinematic framework (material and geometric non-linearities). It is based on a mixed velocity-pressure formulation coming from the virtual energy principle that controls the geometric evolution within each time step. In addition, we deal with the treatment of unsteady contact conditions between the material and the dies. We describe the developments to integrate the boundary contact condition in a mixed formulation. The corresponding inequality condition is set as a linear complementary problem. The mechanical problem is also rewritten under the non-penetration constraint using the penalty method. In section 3, we consider a finite element method to approximate the problem using independent meshes within each body. In section 4, we implement the master-slave algorithm for bodies discretized by linear tetrahedra. Then we focus on iterative methods used to solve the coupled mechanical problem. The next section is devoted to the studies of two academic examples. With these examples, we study convergence and numerical behavior of the algorithm.

2. Setting of the mechanical problem and weak formulation

We consider the deformation of two viscoplastic bodies occupying in the initial configuration two subsets $\overline{\Omega}^l$ of the space $I\!R^3$, $l = 1, 2$. The boundary $\partial\Omega^l$ of the domain Ω^l is assumed to be smooth enough and consists of $\Gamma_v^l, \Gamma_\sigma^l$ and Γ_c. We assumed that the body is submitted to surface forces on Γ_σ^l. The inertia and body forces are negligible. On Γ_v^l, the velocity fields v^l are prescribed. Γ_c denotes contact surfaces with a rigid or a deformable body. The mathematical model of the mechanical problem is based on the constitutive law and the contact condition with friction. In continuum mechanics, viscoplastic flows, involved for instance in hot metal forging, are well modelled by a velocity formulation [SUR 86a].

2.1. *Constitutive equation*

For a viscoplastic constitutive law, the stress deviator s can be derived from a potential ϕ_r, itself a convex function of the strain rate $\dot{\varepsilon}$:

$$s = \frac{\partial \phi_r}{\partial \dot{\varepsilon}} \qquad [1]$$

The strain rate tensor $\dot{\varepsilon}$ is derived from the velocity field in the usual way:

$$\dot{\varepsilon} = \frac{1}{2}(\nabla v +^T \nabla v) \qquad [2]$$

where the superscript "T" indicates a transposition.
The most popular isotropic viscoplastic law is the Norton-Hoff one. It is well suited

for modelling the metal behavior under hot conditions. The function ϕ_r is then given by the following expression:

$$\phi_r(\dot{\varepsilon}) = \frac{K}{m+1}(\sqrt{3}\dot{\bar{\varepsilon}})^{m+1} \quad [3]$$

where K is the material consistency, which can be function of the total strain and the temperature, m is the strain rate sensitivity index and $\dot{\bar{\varepsilon}}$ is the generalized strain rate:

$$\dot{\bar{\varepsilon}} = \sqrt{\frac{2}{3}\sum_{i,j}\dot{\varepsilon}_{ij}^2} \quad [4]$$

The stress deviator can be deduced from (1) and (3):

$$s = 2K(\sqrt{3}\dot{\bar{\varepsilon}})^{m-1}\dot{\varepsilon} \quad [5]$$

If we consider only dense materials, we must take into account the incompressibility condition and write:

$$\text{div}\, v = 0 \quad [6]$$

where the symbol "div" denotes the divergence operator.

Both inertia and gravity effects can be neglected, thus the equilibrium equation is simply:

$$\text{div}\, \sigma = 0 \quad [7]$$

where σ is the stress tensor which can be decomposed into its spheric (hydrostatic pressure), p, and deviatoric, s, parts:

$$\sigma = s - p\text{I} \quad [8]$$

2.2. *Boundary conditions*

Boundary conditions in metal forming processes are important [SOY 92]as they govern the material flow. To the equilibrium equations (6–7) must be added, for any point x of the boundary Γ_c, the conditions resulting from the contact between the bodies sharing Γ_c and between deformable bodie and rigid tools.

The constitutive equations for contact and friction must express both the non-penetration and the possibility of relative sliding associated with a tangential friction shear stress.

2.2.1. *Non-penetration condition*

The unilateral contact condition is decomposed into a geometric condition and a stress condition. In fluid mechanic, this corresponds to no-miscible and no-adhesion conditions. The first one prevents the body Ω^1 to penetrate through the other one Ω^2:

$$(v^1 - v^2) \cdot n \leq 0 \tag{9}$$

where v^1 and v^2 are the velocity fields of the bodies Ω^1 and Ω^2 respectively at any point on the interface Γ_c, n is the outward unit normal to the body Ω^2 at the considered point. (see figure 1).

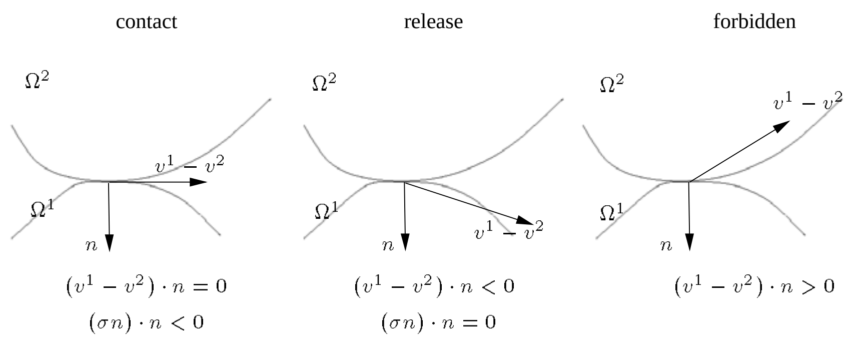

Figure 1. *Unilateral contact condition*

The second condition expresses that, on the contact boundary, the normal stress must be compressive:

$$(\sigma n) \cdot n \leq 0 \quad \text{on } \Gamma_c \tag{10}$$

In other words, there is no adhesion between the two bodies.

The unilateral contact condition is summarized, on Γ_c, by:

$$\begin{cases} \text{if } (\sigma n) \cdot n < 0 & \text{then} \quad (v^1 - v^2) \cdot n = 0 \\ \text{if } (\sigma n) \cdot n = 0 & \text{then} \quad (v^1 - v^2) \cdot n \leq 0 \end{cases} \tag{11}$$

2.2.2. *Friction law*

For a contact point, the viscoplastic friction behavior can be described by a non-linear relation between the shear stress τ at the bodies interface and the tangential sliding velocity v_g. The sliding velocity is defined by:

$$v_g = (v^1 - v^2) - \left[(v^1 - v^2) \cdot n\right] n \qquad [12]$$

The shear stress τ is derived from a convex friction potential ϕ_f:

$$\tau = \frac{\partial \phi_f}{\partial v_g}(v_g) \qquad [13]$$

For hot metal forming application, the Norton law [MOR 70] is often used without any plasticity criterion [CHE 92]. In this context, the function ϕ_f is:

$$\phi_f(v_g) = -\frac{\alpha K}{q+1} ||v_g||^{q+1} \qquad [14]$$

where α is a friction coefficient, q is the sensitivity to the sliding velocity v_g and K is the consistency of the “softest” body. The interface shear stress is deduced from (13) and (14):

$$\tau = -\alpha K ||v_g||^{q-1} v_g \qquad [15]$$

2.3. ***Variational formulation***

2.3.1. *Mechanical problem for viscoplastic body*

Under the hypothesis made above, the weak form of the equilibrium equation (7) can be obtained by the minimization of a dissipation function, with respect to velocity, under the incompressibility condition, on each domain Ω^l for $l = 1, 2$:

$$\phi(v^l) = \min_{\substack{v \in \mathcal{V}(\Omega^l) \\ \text{div}\, v = 0}} \left(\int_{\Omega^l} \phi_r(\dot{\varepsilon})\, dx \right) \qquad [16]$$

where $\mathcal{V}$ is the definition space of the velocity field [BAR 90]. In order to solve the constrained problem (16), it is usual to introduce Lagrange multiplier and transform the minimization problem (16) into the saddle point problem:

$$\max_{p\in\mathcal{L}^2(\Omega^l)} \min_{v\in\mathcal{V}(\Omega^l)} \left(\int_{\Omega^l} \phi_r(\dot\varepsilon)\,dx - \int_{\Omega^l} p\,\mathrm{div}\,v\,dx\right)$$

where $\mathcal{L}^2(\Omega^l)$ is the classical Lebesgue space of square integrable functions.
We now have to find a pair (v^l, p^l) solution of the variation system:

$$\begin{cases} \int_{\Omega^l} s^l : \dot\varepsilon^*\,dx - \int_{\Omega^l} p^l\,\mathrm{div}\,v^*\,dx = 0 \\ -\int_{\Omega^l} p^*\,\mathrm{div}\,v^l\,dx = 0 \end{cases} \qquad [17]$$

for all $(v^*, p^*) \in \mathcal{V}(\Omega^l) \times \mathcal{L}^2(\Omega^l)$.
It can be easily verified that the Lagrange multiplier is the hydrostatic pressure [BRE 91]. Obviously, the second equation of (17) expresses the weak form of the incompressibility condition (6). The existence of a saddle point (v^l, p^l) is discussed in [BAR 90].

2.3.2. *Coupled mechanical problem*

In order to obtain the variational formulation of the coupled mechanical problem between the bodies Ω^1 et Ω^2, we denote $v = (v^1, v^2)$ a vector field of the product space $\mathcal{V} = \mathcal{V}(\Omega^1) \times \mathcal{V}(\Omega^2)$ and $p = (p^1, p^2)$ a scalar of the product space $\mathcal{Q} = \mathcal{L}^2(\Omega^1) \times \mathcal{L}^2(\Omega^2)$. Then the appropriate closed convex set $\mathcal{S}$ of admissible velocity fields is contained in $\mathcal{V}(\Omega^1)\times\mathcal{V}(\Omega^2)$ and incorporates the unilateral contact condition:

$$\mathcal{S} = \{v = (v^1, v^2) \in \mathcal{V}(\Omega^1) \times \mathcal{V}(\Omega^2),\ (v^1 - v^2)\cdot n \le 0 \text{ on } \Gamma_c\}$$

Taking into account the friction condition (14), the virtual work principle is derived from a convex function, similar to (16), for all $v \in \mathcal{S}$:

$$\Phi(v) = \int_{\Omega} \phi_r(\dot\varepsilon)\,dx + \int_{\Gamma_c} \phi_f(v_g)\,ds \qquad [18]$$

$$\Phi(v) = \min_{\substack{v' \in \mathcal{S} \\ \mathrm{div}\,v' = 0}} \Phi(v') \qquad [19]$$

where $\Omega = \Omega^1 \cup \Omega^2$ and the interface $\Gamma_c = \partial\Omega^1 \cap \partial\Omega^2$.
We must underline that we have assumed that the friction zone is fixed. According to (15), the friction condition does not depend on the contact pressure but only on the geometrical position of the material point.

Equation (19) is another minimization problem with now two constraints: the first one is related to the incompressibility condition and the second one to the non-penetration condition which is an inequality constraint. As mentioned before, a Lagrange multiplier, the hydrostatic pressure, is introduced to impose the incompressibility condition.
Unilateral contact condition is an inequality constraint. We remark that inequality can be converted into equality by:

$$(v^1 - v^2) \cdot n \leq 0 \Leftrightarrow \left[(v^1 - v^2) \cdot n\right]^+ = 0 \qquad [20]$$

where $[x]^+$ is the positive part of the quantity x:

$$[x]^+ = \begin{cases} x & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases} \qquad [21]$$

This equality constraint is then imposed by a penalty formulation. Therefore, a new function is defined to handle the different constraints of the problem:

$$L_\rho(v,p) = \Phi(v) - \int_\Omega p \operatorname{div} v \, dx + \frac{\rho}{2} \int_{\Gamma_c} \left[(v^1 - v^2) \cdot n\right]^{+\,2} ds$$

where ρ is the penalty coefficient. The solution (v,p) satisfies the saddle point condition:

$$L_\rho(v,p) = \max_{p' \in \mathcal{Q}} \min_{v' \in \mathcal{V}} L_\rho(v',p') \qquad [22]$$

It can be shown that, when ρ tends towards infinity, the term $-\rho\left[(v^1 - v^2) \cdot n\right]^+$ tends to the contact pressure.
The optimal condition reproduces (weakly) the whole set of equations governing the coupled mechanical problem with frictional unilateral contact:

$$\begin{cases} \displaystyle\int_\Omega s : \dot{\varepsilon}^* \, dx - \int_\Omega p \operatorname{div} v^* \, dx - \int_{\Gamma_c} \tau \cdot v^* \, ds \\ \qquad \displaystyle + \rho \int_{\Gamma_c} \left[(v^1 - v^2) \cdot n\right]^+ v^* \cdot n \, ds = 0 \\ \displaystyle - \int_\Omega p^* \operatorname{div} v \, dx = 0 \end{cases} \qquad [23]$$

for all $(v^*, p^*) \in \mathcal{V} \times \mathcal{Q}$.

3. Discretization

The material flow studied is an evolution problem. A double discretization must be considered:

– a time discretization,

– a space discretization.

3.1. *Time integration*

The material movement is discretized in time and approached by succession of discretized configurations (see figure 2).

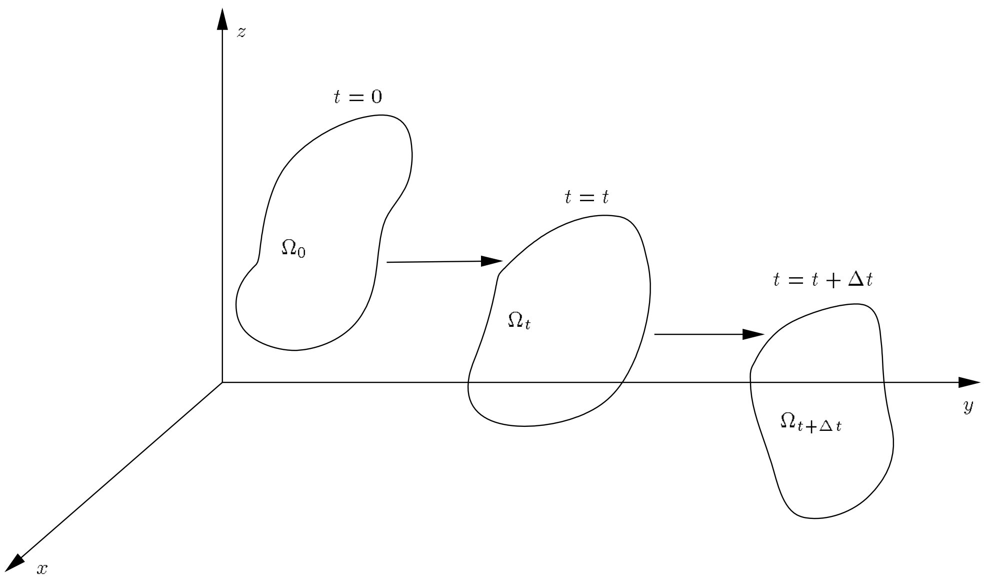

Figure 2. *Time discretization movement of a domain* Ω

This kinematic representation is modelled by an updated Lagrangian description. Instead of locating the body in relation to the initial configuration Ω_0, it is located in relation to the configuration at time t, Ω_t, to determine the next configuration at time $t + \Delta t$, $\Omega_{t+\Delta t}$. For each configuration, the equilibrium equations must be satisfied.

For purely viscoplastic materials, the most popular scheme is a finite difference scheme: the explicit Euler scheme. Knowing the domain $\Omega = \Omega^1 \cup \Omega^2$ at time t, the velocity v_h^t and pressure p_h^t fields are calculated by solving the discretized equations of the quasi-static problem (23) at time step t. The material update is then performed according to:

$$x^{t+\Delta t} = x^t + \Delta t\, v^t \qquad [24]$$

This scheme is simpler compared to a semi-implicit second order scheme [SUR 86b] or an explicit second order Runge-Kutta scheme [BOY 99].

3.2. *Incremental contact condition*

The non-penetration condition (9) must also be discretized in time.

On a given configuration, at time t, in order to check the relative position of a material point of coordinate vector x^t with respect to a surface of another body, called master surface, we define the following gap function g:

$$g(x^t, t) = \left(x^t - \pi(x^t)\right) \cdot n^t \qquad [25]$$

where $\pi(x^t)$ is the orthogonal projection of x^t on the master surface, and n^t is the outward unit normal to the current master body (see figure 3).

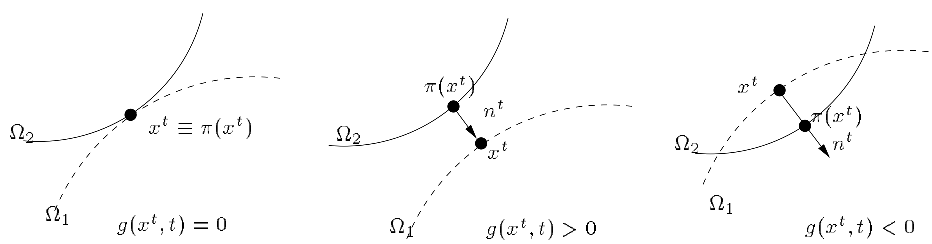

Figure 3. *Definition of the gap function*

The non-penetration condition is equivalent to:

$$g(x^{t+\Delta t}, t + \Delta t) \geq 0 \qquad [26]$$

With an explicit time integration scheme (24), a first order Taylor expansion of (26) provides the explicit incremental contact condition:

$$g(x^{t+\Delta t}, t + \Delta t) \simeq g(x^t, t) - \left(v^1(x^t) - v^2(\pi(x^t))\right) \cdot n^t \Delta t$$

REMARK. — In this paper, we neglect the variation of the normal n during the incremental time Δt, because, in our application, the master body undergoes small deformation compared to the slave body.

The non-penetration condition is then rewritten as:

$$(v^1(x^t) - v^2(\pi(x^t))) \cdot n^t - \frac{g(x^t, t)}{\Delta t} \leq 0 \qquad [27]$$

Therefore, we can combine the velocity formulation with the incremental formulation. For each configuration, the integral formulation (22) becomes:

$$\begin{aligned} L_\rho(v,p) &= \Phi(v) - \int_\Omega p \operatorname{div} v \, dx \\ &+ \frac{\rho}{2} \int_{\Gamma_c} \left[(v^1 - v^2) \cdot n - \frac{g}{\Delta t}\right]^{+^2} ds \end{aligned} \qquad [28]$$

3.3. *Space finite element discretization*

We suppose that Ω^l, $l = 1, 2$, are domains with polygonal boundaries. With each body Ω^l, we associate a mesh T_h^l, made of tetrahedra denoted κ. Therefore, we can write:

$$\overline{\Omega}^l = \bigcup_{\kappa \in T_h^l} \overline{\kappa}$$

A spatial discretization is performed with isoparametric elements. The discrete velocity field v_h is expressed in terms of the shape functions N_n and the nodal velocity vectors V_n as

$$v_h = \sum_n V_n N_n(\xi) \qquad [29]$$

with the parametric relation between the physical coordinate vector x_h and the natural coordinate in the reference space ξ, using the nodal coordinate vector X_n:

$$x_h = \sum_n X_n N_n(\xi) \qquad [30]$$

We can then write the strain rate tensor with the usual B operator:

$$\dot{\varepsilon}_h = \sum_n V_n B_n(\xi) \qquad [31]$$

where each component of the strain rate tensor is defined by, $i, j = 1, \cdots, 3$:

$$\begin{aligned} \dot{\varepsilon}_{h,ij} &= \frac{1}{2} \sum_n \left(V_{n,i} \frac{\partial N_n}{\partial \xi_j} + V_{n,j} \frac{\partial N_n}{\partial \xi_i} \right) \\ &= \sum_{n;k=i,j} V_{n,k} B_{n,k}(\xi) \end{aligned} \qquad [32]$$

In the mixed formulation of the mechanical problem (23), compatible shape functions M_m must be selected for the pressure field p_h [BRE 74], which is expressed in terms of the discrete pressure parameters P_m:

$$p_h = \sum_m P_m M_m(\xi) \qquad [33]$$

In our case, a slight simplification of the so-called mini-element [COU 96], first introduced by [ARN 84], is used and leads to a stable mixed finite element formulation. Denoting by $I = \{i,\, i = 1, \cdots, q\}$ the set of indexes i such that X_i belongs to a potential conatct zone, is a nodal point, the convex set $\mathcal{S}$ can be approximated by:

$$\begin{aligned} \mathcal{S}_h &= \{v_h = (v_h^1, v_h^2) \in \mathcal{V}_h = \mathcal{V}_h(\Omega^1) \times \mathcal{V}_h(\Omega^2), \\ h_i &= \left(V_i^1 - v_h^2(\pi(X_i)\right) \cdot n - \frac{g(X_i)}{\Delta t} \leq 0,\, i \in I\} \end{aligned}$$

where $V_i^1 = v_h^1(X_i)$.

Considering a nodal contact formulation, a simple discrete form of (28) can be written as:

$$L_{h,\rho}(v_h, p_h) = \Phi(v_h) - \int_\Omega p_h \operatorname{div} v_h \; dx + \frac{\rho}{2} \sum_{i=1}^{q} |\Gamma_{ci}| [h_i]^{+2} \qquad [34]$$

where $|\Gamma_{ci}|$ is a weighting surface for the node i. It represents one third of the surface of the triangles containing i.

Then, the minimization problem is to find $(v_h, p_h) \in \mathcal{V}_h \times \mathcal{Q}_h$ such that:

$$L_{h,\rho}(v_h, p_h) = \max_{p' \in \mathcal{Q}_h} \min_{v' \in \mathcal{V}_h} L_{h,\rho}(v', p') \qquad [35]$$

where $\mathcal{Q}_h \subset \mathcal{L}^2(\Omega^1) \times \mathcal{L}^2(\Omega^2)$.

From the continuous standpoint, the interface Γ_c represents the same surface for both bodies, but they are not the same after discretization. If the meshes of the bodies coincide, the calculation of the solution of the problem is relatively straightforward, using the same interface nodes and surface elements. The coupled mechanical problem with frictional contact is then written as a non-linear system of equations $\mathbf{R}(v_h, p_h) = 0$, the components of which are expressed by:

$$\left\{ \begin{array}{l} \displaystyle\sum_{l=1}^{2} \left\{ \int_{\Omega^l} 2K(\sqrt{3}\dot{\bar{\varepsilon}}_h^l)^{m-1} \dot{\varepsilon}_h^l : B_{n,k}\, dx - \int_{\Omega^l} p_h^l \mathrm{trace} B_n \, dx \right\} \\ \displaystyle + \int_{\Gamma_c} \alpha K ||v_g||^{q-1} v_{g,k} N_n \, ds \\ \displaystyle + \rho \sum_{i=1}^{q} |\Gamma_{ci}| [h_i]^+ n_k^i = 0 \qquad \forall n, \, \forall k \\ \displaystyle - \sum_{l=1}^{2} \int_{\Omega^l} M_m \mathrm{div}\, v_h^l \, dx = 0 \qquad \forall m \end{array} \right. \qquad [36]$$

where n^i is the normal at the node x_i potentially in contact and outward with respect to the body Ω^2.

However, in general case, the meshes do not match on the contact surface. The difficulty is to impose the friction and the non-penetration condition.

4. Master-slave algorithm

The friction and contact conditions are written in a discretized manner between the velocity degrees of freedom of the coupled mechanical problem. With each body Ω^l, we associate a family of triangulations T_h^l, made of tetrahedra κ. The contact zone Γ_c inherits two independent families of discretizations arising from T_h^1 and T_h^2. The mesh $\mathcal{T}_{c,h}^l$ on Γ_c is defined as the set of all the faces of $\kappa \in T_h^l$ on the contact zone. The set of nodes associated with $\mathcal{T}_{c,h}^l$ is denoted χ_h^l. In general χ_h^1 and χ_h^2 are not identical on account of the non-matching meshes.

In order to express the contact constraint, we need to define a new discrete set $\mathcal{S}_h$ of the kinematic admissible velocity fields.

We chose to use the so-called master-slave algorithm: one of the bodies is the slave and the other one the master. In literature, a first approach consists in expressing the local contact conditions as nodal contributions [CHA 86, WRI 90, VAU 98, CHE 98, HIL]. An other way is to consider the discrete contact conditions at the quadrature points [CHA 88, LAU 93, PAN 00]. Here, we extend the nodal implementation to a 3D application with tetrahedric meshes. The contact algorithm will ensure that slave nodes will not penetrate into master faces. The non-penetration condition will be written between a slave node and a master triangular face in the current configuration. At the beginning of each time increment, a search algorithm is performed to build

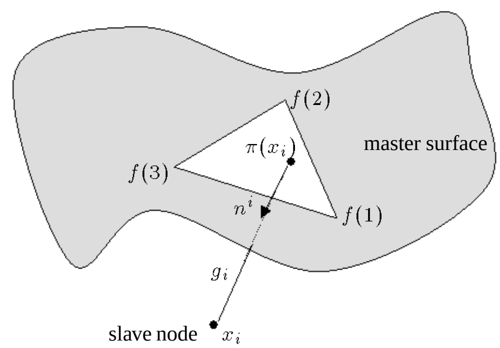

Figure 4. *Master-slave pair*

the node-face contact pairs which determines for all slave boundary nodes the closest master face. To speed up the search, the master faces are organized into a hierarchical tree.

4.1. *The contact term*

A closest point projection is used to determine the gap between the slave node and the master face. If the gap between the slave node and the master face is within a given tolerance, the slave node is considered to be potentially in contact. Denoting by $I = \{i \,:\, i = 1, \cdots, q\}$ the set of indexes such that $X_i \in \chi_h^1$ is a slave nodes potentially in contact with a master face $f \in \mathcal{T}_{c,h}^2$, the nodal non-penetration kinematic condition becomes:

$$\left(V_i^1 - \sum_{m=1}^{3} \xi_m V_{f(m)}^2 \right) \cdot n^i - \frac{g_i}{\Delta t} \leq 0 \quad \forall i \in I \qquad [37]$$

where (see figure 4):

– V_i^1 is the velocity field at the slave node X_i (on the boundary of the body Ω^1),

– $f(m)$ is the mth node of the master face f,

– $V^2_{f(m)}$ is the velocity field at the mth mode of the master face (on the boundary of the body Ω^2),

– ξ_m are the parametrical coordinates of the projection $\pi(X_i)$ of the slave node X_i onto the master face,

– n^i is the outward normal vector to the master face f at the projection node $\pi(X_i)$,

– g_i is the gap between the slave node X_i and its projection onto the master face at the beginning of the increment.

V_i^1 and $V^2_{f(m)}$ are unknowns of the problem.

Therefore, the penalty term of the system (36) can be rewritten as:

$$\rho \sum_{i=1}^{q} |\Gamma_{ci}| \left[\left(V_i^1 - \sum_{m=1}^{3} \xi_m V^2_{f(m)} \right) \cdot n^i - \frac{g_i}{\Delta t} \right]^+ n_k^i \; \forall k \qquad [38]$$

In the same way, the sliding velocity is redefined, for all slave node X_i:

$$v_g(x_i) = \left(V_i^1 - \sum_{m=1}^{3} \xi_m V^2_{f(m)} \right) - \left[\left(V_i^1 - \sum_{m=1}^{3} \xi_m V^2_{f(m)} \right) \cdot n^i \right] n^i$$

4.2. *The friction term*

The velocity is known for a node X_i and for his projection on the master body. In order to simplify the integral term associated to the friction law in the system, the integral (36) is then approximated by a nodal contribution:

$$\sum_{i=1}^{q} -\alpha K^1 |\Gamma_{ci}| ||v_g(x_i)||^{q-1} v_g(x_i)_k \quad \forall k \qquad [39]$$

The friction term is considered constant on surface $|\Gamma_{ci}|$ related to the node i on the boundary. In classical hot forging simulation this approximation has been validated by comparing nodal and integral formulations.

5. Iterative solver

The mechanical problem of deformable bodies coupled by the frictional contact problem is solve simultaneously in all domains. The material behavior (viscoplastic and incompressible) and the unilateral contact condition lead to the solution of a highly non-linear multi-variable problem. At each time step, the large non-linear system (36) is solved by a quasi-Newton method:

1) (v_h^{k-1}, p_h^{k-1}) given, find $(\Delta v_h^k, \Delta p_h^k)$ such as:

$$\frac{\partial \mathbf{R}}{\partial (v_h, p_h)}(v_h^{k-1}, p_h^{k-1}) \cdot (\Delta v_h^k, \Delta p_h^k) = -\mathbf{R}(v_h^{k-1}, p_h^{k-1}) \qquad [40]$$

2) Update velocity and pressure fields:

$$v_h^k = v_h^{k-1} + \lambda \Delta v_h^k$$
$$p_h^k = p_h^{k-1} + \lambda \Delta p_h^k$$

where λ is a linear search coefficient.

3) Check convergence:

if $\|v_h^k - v_h^{k-1}\| + \|p_h^k - p_h^{k-1}\| \leq \epsilon$
then STOP
else $k \leftarrow k+1$ and go to 1.

k indicates the Newton iteration number.

An iterative solver is used for each linear sub-system (40). The solver [COU 97b] has been first proposed for calculation of part deformations in forging process. It is mainly based on preconditioned conjugate residual method as it has been suggested in [WAT 93]. This solver was successfully improved in the global multi-bodies system triggered by coupled mechanical formulation for part and dies deformations calculation.

The convergence rate of iterative solver depends strongly on the conditioning of the stiffness matrix which is affected by penalty terms. The use of efficient preconditioners can improve this shortcoming and provides a robust solver for the coupled problem. Our formulations lead to block dominant stiffness matrix. Here, a block diagonal preconditioning [COU 97b] will be sufficient to capture the main matrix coefficients possibly augmented by the penalty terms and preserve the nice convergence rate of the solver. To capture all potentially augmented terms, some of which are located not near the diagonal of the matrix, incomplete Cholesky preconditioner [PER 00] can be used instead.

This fully coupled approach is expensive in CPU time but provides a strong coupling between the bodies via the contact and friction terms. The equilibrium equations are solved for the whole system. Finally the main advantage of such an approach is that the parallelization of the software is simpler than with an iterative coupling because the formulation is very similar to the mono-domain one. The main problem is to manage with the different boundary through the processors.

6. Examples

In this section, we present some numerical examples to illustrate multi-bodies mechanical calculation. The proposed model has been implemented in the finite element code Forge3®, which until now predicts only the part flow.

6.1. *Block upsetting*

The coupled mechanical algorithm has been first applied to the flattening of a block in order to estimate the computational time. The symmetries of the problem allow us to simulate one quarter of the process. The geometry of the problem is composed of three bodies as shown in figure 5 . A constant vertical velocity is imposed on the top of the upper die Ω_2. No displacement is imposed to the bottom of the lower die Ω_3. At the interface of the bodies, the contact is supposed to be sliding with a viscoplastic friction law defined by $\alpha = 0.2$ and $q = 0.139$. Other boundaries are free surfaces. We choose a consistency of 179.2 MPa and a strain rate sensitivity $m = 0.139$ for the intermediate block. The lower and upper dies have a consistency one hundred times higher. These values are representative of hot forging of steel.

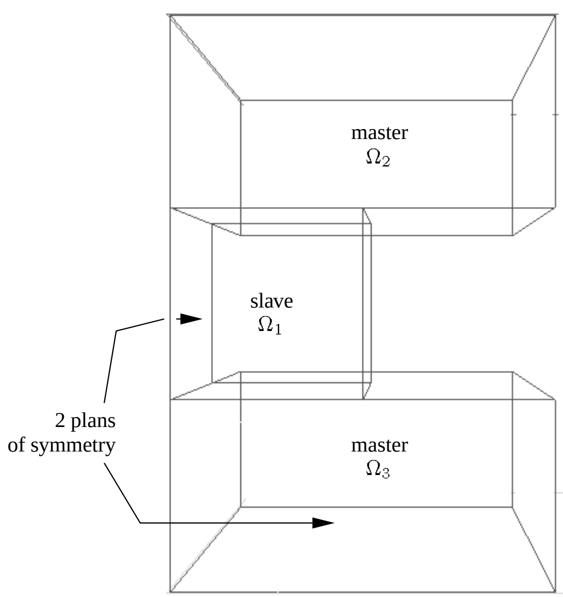

Figure 5. *Geometric description*

The intermediate block is the slave body and the upper and lower dies are master bodies. In our calculation, the slave mesh is always finer than the master one. This choice is made according to observations in bi-dimensional cases [HIL , HAB 92, HAB 97]. Furthermore, the interface meshes do not coincide.

In order to study the asymptotic convergence of the numerical calculation for one time step, we build a family of finer and finer meshes for both bodies. We begin with very coarse meshes. The edges length ratio between the coarsest and the finest mesh is about ten. The iterative solver with a block diagonal preconditioner is used. Figure 6 shows the computation times as a function of the total nodes number (part nodes + dies nodes). The asymptotic behavior is proportional to (nb. nodes)$^{1.48}$ (the asymptote equation is: $0.1167 + 4.0210^{-4}\times$ (nb. nodes)$^{1.48}$). The convergence rate is similar to the theoretical rate which is (nb. nodes)$^{1.5}$ [COU 97b]for a single body. It is not affected by non diagonal penalty terms. That means that if we increase the number of degrees of freedom by three, adding the mechanical computation in the tool, we increase the CPU time of approximatively 5.2. This shows the robustness of the method compared to other iterative methods where balance equations are solved independently on each body.

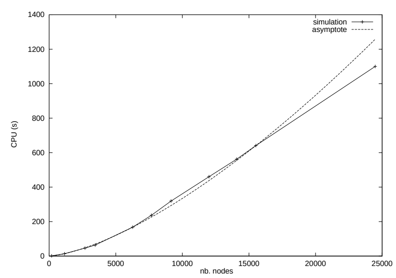

Figure 6. *Asymptotic convergence*

Figure 7 shows the end of the compression and the evolution of the contact area. The convex form of the free surface is quite similar in multi-bodies calculation and in single body calculation. We can observe that the non-penetration condition is well respected during the process.

6.2. *Hemispherical punching*

The punching problem has been studied to qualify the contact algorithm. The symmetries of the problem allow us to simulate a sixteenth of the process. The geometry of the problem are shown in figure 8. The upper plane of the punch is submitted to a constant vertical velocity. We choose a consistency of 169.5 MPa and a strain rate sensitivity $m = 0.139$ for the part. The punch has a consistency one hundred times higher. At the interface of the bodies, the contact is supposed to be sliding with a viscoplastic friction law defined by $\alpha = 0.4$ and $q = 0.139$. The part is the slave body and the punch the master one.

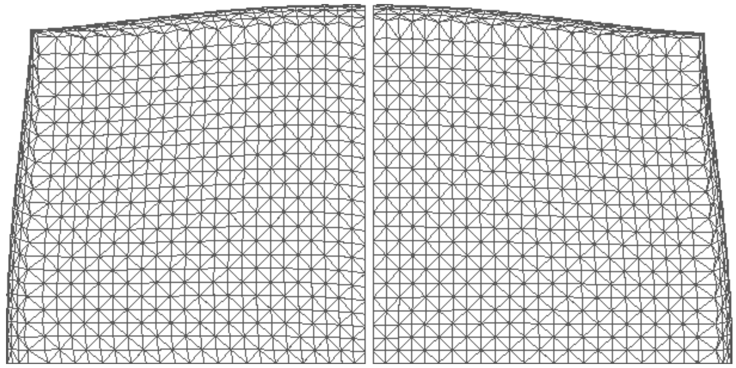

Figure 7. *Upper view of the intermediate block with multi-bodies calculation (left) and with single body calculation (right)*

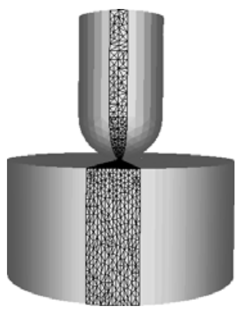

Figure 8. *Geometric description*

In this calculation, remeshing is allowed. Optimal topology method for tetrahedra is used to generate automatically a better mesh, when degeneracy of elements occurs [COU 97a, COU 00]. This is important for large-strain problems in which the mesh degenerates long before the end of the simulation. The algorithm improves the mesh, and the nodal velocity fields are interpolated from the old to the new mesh. In this study only the slave body is remeshed, while the other remains unchanged throughout a given simulation. Using the remeshing algorithm does not deteriorate the satisfaction of the contact conditions because the possible penetrations of the slave domain into the master ones are small enough to be corrected. The penetrations of the master domain into the slave one are of the same magnitude as the ones dued to the master slave approach.

The required number of Newton-Raphson iterations for convergence is seven on average. The convergence test requires a relative convergence of 10^{-6} on the residual norm. This residual obviously takes into account the contact terms described in equation (38). The explicit contact algorithm however can produces artificial small penetrations that have to be corrected at the next time increment. The order of magnitude of the penetration is about 0.1 mm. The edges length near the contact zone for the slave body is in order of 10 mm.

In this example, it is clearly shown that, due to constitutive or geometric nonlinearities, at each step, the updated finite element meshes cannot fit together on the contact zone. In other words, mesh adaptivity procedures used lead to non-matching meshes. The initial and deformed configurations are depicted in figure 9. Figure 10 represents the initial and the deformed meshes near the contact zone, and we observe a deformed configuration which seems quite satisfactory, particularly on the contact part. The chosen algorithm solving discrete contact condition for non matching meshes is quite efficient.

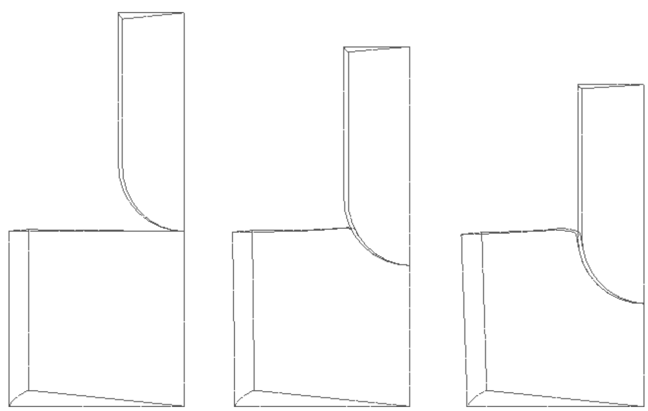

Figure 9. *The initial and deformed configurations*

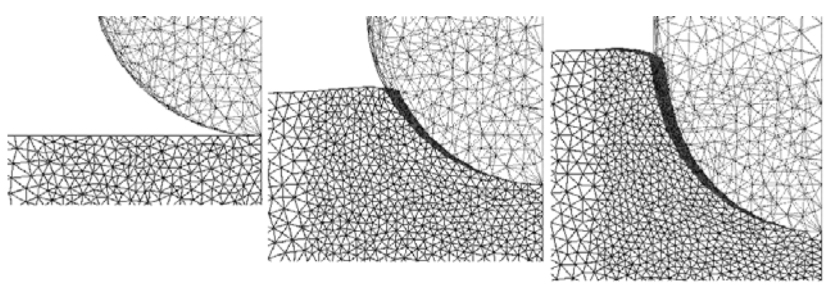

Figure 10. *The initial and deformed meshes near the contact zone*

7. Conclusion

In order to solve the unilateral contact problem with friction between deformable bodies, we have considered a finite element approximation able to treat non-matching meshes on the contact zone. The basic equations in viscoplastic case and the contact problem have been recalled.

The unilateral contact condition handled by a penalty method and associated with a slave node-master face technique simulates accurately contact between deformable bodies.

The coupled mechanical problem was approximated by a stable mixed velocity-pressure formulation based on a tetrahedric mini-element. An iterative solver is used to solve very large problems originating from the simultaneous resolution of the mechanical problem for all the bodies. The contact terms deteriorate the conditioning of the stiffness matrix. However, the convergence is still verified and with the same rate.

The coupled calculation gives material informations within each body. Here, only the slave body is submitted to large deformations and self-contact is not taken into account. The calculations performed have shown that the contact algorithm is reliable and accurate.

Finally, the extension of such a contact technique to hot forging simulation with elastic tools will be investigated. The selected strategy is very important in view of an easy extension of the present parallel computation approach based on mesh partitioning.

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