# An inventory control model with interconnected logistic services for vendor inventory management

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**Abstract:** *This paper proposes an inventory control model taking advantage of interconnected logistic services in the Physical Internet for fast-moving consumer goods (FMCG) sector. Unlike current hierarchical inventory model where the source of each is pre-assigned, the goods are stored and distributed in an interconnected and open network of PI-hubs which enables storage capacity and transportation sharing among different companies around the network. As a result, theoretically, the suppliers can push their goods all around the network and the retailers can be served by any hub in the network. A non-linear global optimization inventory model to minimize the total logistic costs is proposed and a heuristic using simulated annealing is applied to solve the problem. Numerical experiments are taken to compare the performance of the proposed PI inventory model and classic inventory control model for different settings of a typical supply network. Results suggest that the PI inventory control model can always reduce the total logistic cost while reaching a comparable or improved end customer service level.*

**Keywords:** *Inventory control, Physical Internet, Optimization*

## 1 Introduction

In the past decades, numbers of papers have dealt with inventory problems based on hierarchical inventory systems as inventory often accounts for a large proportion of a company's supply chain management costs. According to (Cachon and Terwiesch 2005), generally speaking in Fast Moving Consumer Goods - FMCG sector, the inventory cost represents averagely up to 40% of total logistics cost, in addition to the cost of shortages in retailers, i.e. around 7% of references in supermarket. Thus, efficiently managing inventory systems becomes quite crucial to improve a company's business performance. Nevertheless the efficiency to reduce inventories, the existing inventory model extensions are based on hierarchical inventory structure, which have been more and more challenged by the new practices of today's supply chains: the increasing uncertainty of customer demand, increasing service requirements reflected in the importance of narrow delivery time windows and pressure on lead time reduction, and also longer lead time from sourcing points to end customers due to the delocalization of warehouses (Angel et al. 2006). Assuming that the hypothesis of hierarchical inventory systems is an inherent limit for current inventory models, we are interested here to study innovative models that are more open, flexible and horizontally collaborative.

In this paper, we propose an inventory control model with the recently proposed logistic concept - Physical Internet with interconnected logistic services and investigate its impacts compared to current inventory control models to different logistic network configurations. Inspired by the metaphor of Digital Internet, the Physical Internet (PI) aims to integrate heterogeneous and independent logistics networks into an open and interconnected global system through standard containers and routing protocols (Ballot and Montreuil 2014). As an innovative concept in logistics, further studies about the concept of the network are carried out by (Montreuil 2011; Ballot, Montreuil, and Fontane 2011a; Montreuil, Meller, and Ballot 2013). (Sarraj et al. 2014) study the transportation performance of PI network in terms of FMCG cases in France and assess the new organization can reduce up to 35% of actual transportation cost through the optimization of full truckload and integration of different transportation means.

Concerning inventory problems in the Physical Internet, (Pan et al. 2014) is the first to describe the inventory problems in the Physical Internet and investigate the first perspectives of PI inventory models compared to classic inventory models. Instead of classical centralized and hierarchical storage organizations in current logistic systems, the PI network enables a distributed storage of goods in an open and interconnected network of hubs. The network of hubs may be managed by the Logistic Service Providers (LSP) and shared by companies including other suppliers and their customers (retailers). In other words, theoretically, each supplier is able to store their goods at any hub all around the network and each customer (retailers) can be served by any hub or directly by the suppliers, resulting in more supply options and increasing service level for retailers and potential reduced logistic costs by sharing of storage capacity and transportation means to supply the demand. The aim is to make current inventory systems more open, robust, and economic thus more sustainable. As a result, the sourcing points for each replenishment order are no longer pre-determined and can be dynamically decided according to source selection strategies and needs. Thus under this structure, the inventory decisions for vendors lie on: 1) how to respond to each replenishment order from retailers with required constraints; 2) how to push inventories in the PI-network to satisfy these orders with certain objectives.

To gain insight into the question, we study a single FMCG (fast moving consumer good) product inventory problem in a Physical Internet network of hubs supplying a group of retailers which face normal distributed end-customer demands. We adopt a centralized vendor-decision framework for the supplier who determines the inventory control policies for the warehouses and DCs or the hubs. The objective is to determine optimal inventory control policies of hubs to minimize the total cost of the distribution system while satisfying replenishment orders from retailers. We suppose the optimal replenishment parameters at retailers are obtained locally to minimize its own total cost and considered as input information for the distribution network. To solve the problem, a global nonlinear optimization model is proposed and a heuristic using simulated annealing is applied. A simulation study is taken to validate the optimized inventory control policies. Different settings of the typical network are analyzed. Our results suggest that the PI-inventory model with dynamic source selection strategies can significantly reduce the supply chain management cost compared to classic centralized inventory control strategy while reaching a similar or improved end customer service level at retailers (defined as the percentage of customer satisfied by inventory on hand).

The remainder of this paper is organized as follows. In Section 2, we discuss the related works in the literature. In Section 3, the optimization model developed to PI inventory problem will be presented. Then, in Section 4 the optimization model will be implemented in case studies of FMCG chains. A number of scenarios are proposed and studied in order to validate the model and study the pertinence of model in different configuration of network. Finally, Section 5 concludes this paper by giving some perspectives to the next works.

## 2 Literature Review

Within the literature we found the following three inventory control models based on current hierarchical inventory system close to the PI inventory modality: i) inventory models with lateral transshipments that allow inventory movements among members of the same echelon; ii) inventory models with multi-sourcing options which enables a replenishment order to be satisfied by multiple supplying points; iii) inventory routing problems which combines vehicle routing and inventory control problems and where a supplier decides when to visit its customers, how much to deliver to each of them and how to combine distribution flows into vehicle routes.

Motivated foremost by the aim of reducing lead times, lateral transshipment refers to stock movements between the same echelon locations within an inventory system. Recent comprehensive overviews are provided by (Paterson et al., 2011). Two types of transshipments are often addressed according to the timing of the transshipments: 1) reactive transshipments in response to an existing stock-out as seen in (Krishnan and Rao, 1965; Robinson, 1990; Olsson, 2010); 2) proactive transshipments to prevent the future stock-out, as seen in (Gross, 1963; Diks and De Kok, 1998; Tagaras and Vlachos, 2002). The literature has shown that transshipment is quite profitable for retailers with long replenishment lead times from suppliers and who are located closer to one another or who have grand shortage penalty cost. In spite of the horizontal sharing of inventories in both transshipments and PI inventory model, the hubs in the Physical Internet are fully interconnected and the source selection is dynamically determined while in the lateral transshipments the source is pre-assigned and the transshipments are used as a support to regular replenishment orders.

The research of the multi-sourcing inventory model can be divided into two categories according to whether an order can be split into sub-quantities and met by several source supplying points: 1) without order splitting which focus on source substitution method, as seen in (Ng et al., 2001; Çapar et al., 2011; Veeraraghavan and Scheller-Wolf, 2008); 2) with order splitting which focus on inventory allocation method in addition to source substitution, as seen in (Sculli and Wu, 1981; Ryu and Lee, 2003; Song et al., 2014). The literature shows that the multi-sourcing can reduce the mean and variance of the effective lead time and a split order model always has lower stock levels than the equivalent non-split model. However, the multi-sourcing options in current multi-sourcing models are only restricted between the upper level stocking points and their successive demanding points. The same echelon stocking points are independent and no products flow at the same echelon stocking points are allowed. Therefore we conclude that our model differs from the existing multi-sourcing inventory models in literature.

Similarities are also found in the inventory - routing problems (IRPs). The pioneer contributions of IRPs can be date back to1980s (Bell et al. 1983) and recent reviews are shown in (Bertazzi and Speranza 2012). The existing literature demonstrates that this strategy can improve the supply chain performance by savings on distribution and production cost for suppliers by coordinating shipments to different customers. Despite the fact there may exist vehicle routing optimization in both models, the PI inventory models differ from the IRPs mainly from the following two facts:1) the transportation sharing in the PI network is realized by modularized and standardized containers and routing protocols including vehicle routing optimization, interconnection of multi-modal transport, and etc.; 2) the PI network is able to rebalance stocks in the PI network of hubs to satisfy replenishment orders from retailers while the balance of stocks among different retailer companies is not addressed in IRP.

(Pan et al. 2014) describe inventory problems based on the Physical Internet and propose a rule-based simulation inventory control model with different source selection strategies. However, due to the lack of optimization model, determination of optimal solutions as well as the quantitative study of different settings of the logistic network is not fully addressed in this paper. Hence, there exist no corresponding optimization inventory control model to the principles of proposed PI inventory model. It is a new research issue and a new topic in inventory management.

# 3 Assumptions and formulations

## 3.1 Assumptions

We consider single-product inventory problems of a typical FMCG network with a plant (a vendor) serving a group of $Nr$ retailers through a network of $M$ intermediate stocking points (hubs or DCs or WH). Two types of networks are taken into account: classic centralized hierarchical inventory network and PI inventory network. The Vendor-Managed Inventory (VMI) decision framework is adapted where the vendor accesses inventory information and decides inventory control decisions at intermediate stocking points (hubs or DCs or WH) to minimize the total logistic costs. The retailers face real demand data from the FMCG industrial sector and the optimal inventory control decisions of each retailer are computed locally based the model of (Giard 2005). The results are considered as the input to the optimization model for the vendor. The objective is to find optimal inventory control decisions of intermediate stocking points and source selection decisions to minimize the total logistic costs while satisfying demands from retailers with a similar end customer service level.

The following common assumptions are adapted for all scenarios: a) Each stocking facility including hubs, DCs, WH and retailers applies a (R, Q) continuous review policy; b) The plant is assumed to always have adequate stocks to meet the demands and there is no capacity constraint for hubs; c) Replenishment orders from retailers unmet immediately are considered with a penalty cost; d) The lead times among all the interconnected facilities are given and assumed to be constant; e) The orders are served on a first-come-first-served basis; f) No partial delivery is allowed; g) End customer demands to retailers are uncertain and subject to normal distribution; h) Vendor makes all source selections to supply the retailers; i) The optimal replenishment policies for retailers are determined by the algorithm proposed by (Giard 2005); j) Localizations of all sites remain the same in all scenarios for comparison sake.

Five logistic costs are considered: inventory holding cost, transportation cost, ordering cost, the penalty cost and handling cost. The holding cost are charged for each unit in stock per time unit at hubs or DCs or WH. The transportation cost for each delivery of goods are considered and includes three parts: upstream transportation cost from plants to hubs or WH, upstream transportation cost among hubs or towards DC, and downstream transportation cost to retailers. Each replenishment order placed incurs a fixed ordering cost. Penalty costs for retailers' orders unmet immediately are charged and assumed to be proportional to product value. Finally, handling cost are considered for each movement of stocks enter and leave hubs or DCs or WH.

### 3.2 Notations and formulations

To introduce the optimization model, we use the following notations. And we always refer to Stock Keeping Unit (SKU) to the minimum unit we consider in the model.

Notations:

$M$: set of intermediate stocking points (plant index by 0).

$Nr$: set of retailers.

$T$: configuration time of the inventory system, indexed by $n$ (1 year = 365 days).

$dis_{ij}$: distance between intermediate stocking point $i \in M$ and retailer $j \in Nr$.

$dis_{ki}$: distance between intermediate stocking point $k \in M$ and $i \in M$.

$dis_{0i}$ : distance between intermediate stocking point $i \in M$ and the plant 0.

$(u_j, \sigma_j)$: average and standard deviation of end customer demand at retailer $j$.

$(q_j, s_j)$: replenishment policy of retailer j - $q_j$ for batch size and $s_j$ for reorder point.

$H_i$: daily holding cost per SKU at intermediate stocking point $i$.

hd: handling cost per SKU operated.

$p_j$: daily penalty cost per SKU of unmet orders from retailer $j \in Nr$, proportional to product value.

$c_1$: downstream transportation cost per kilometer per SKU from the hubs to retailer.

$c_2$: upstream transportation cost per kilometer per SKU from the plant to hubs and among hubs.

$A$ : fixed ordering cost per order.

$IL_{in}$ or $IL_{jn}$: inventory level at intermediate stocking point $i$ or retailer $j$ between $n$th and $(n+1)$th day.

The decision variables are:

$R_i$: intermediate stocking point $i$'s reorder point.

$Q_{i0}$: intermediate stocking point $i$'s batch size (order quantity) from the plant.

$Q_i$: intermediate stocking point $i$'s batch size (order quantity) from other intermediate stocking points.

$x_{0in}$: binary variable of whether choose plant 0 to satisfy the demand of intermediate stocking point $i$ at time $n$th day, if so $x_{0in} = 1$, otherwise 0;

$x_{kin}$: binary variable of whether choose intermediate stocking point $k \in M$ to satisfy the demand of intermediate stocking point $i$ $(i \neq k)$ at time $n$th day.

$x_{kjn}$: binary variable of whether choose facility $k \in M$ to satisfy the demand of retailer $j$ at time $n$th day.

The objective function:

$$
\begin{aligned}
Minimize\ C_{tot} &= \left[\sum_{i=1}^{M}\sum_{n=1}^{T} IL_{in}H_i\right]_{(1)} + \left[\sum_{j=1}^{Nr}\sum_{n=1}^{T}\sum_{k=1}^{M} x_{kjn}[c_1 q_j dis_{kj} + q_j * hd]\right]_{(2)} \\
&+ \left[\sum_{i=1}^{M}\sum_{n=1}^{T}\sum_{k=1,k\neq i}^{M} x_{kin}(c_2 Q_i dis_{ki} + A + 2Q_i * hd)\right]_{(3)} \\
&+ \left[\sum_{i=1}^{M}\sum_{n=1}^{T} x_{0in}(c_2 Q_{i0} dis_{0i} + A + Q_{i0} * hd)\right]_{(4)} \\
&+ \left[\sum_{n=1}^{T}\sum_{j=1}^{Nr}\left(1 - \sum_{k=1}^{M} x_{kjn}\right) * u_j * p_j\right]_{(5)}
\end{aligned}
$$

Subject to:

$0 \leq \sum_{k=1}^{M} x_{kjn} \leq 1 \quad \forall j \in Nr, \forall n = 1 \dots T$ (6)

$x_{kjn} \in \{0,1\} \quad \forall k \in M, \forall j \in Nr, \forall n = 1 \dots T$ (7)

$0 \leq \sum_{k=0,k\neq i}^{M} x_{kin} \leq 1 \quad \forall i \in M, \forall n = 1 \dots T$ (8)

$x_{kin} \in \{0,1\} \quad \forall i \in M, \forall k \in M \cup \{0\}, i \neq k, \forall n = 1 \dots T$ (9)

$R_i, Q_{i0}, Q_i$: Integers, $\forall i \in M$ (10)

Where equation (1) represents the total annual holding cost at the hubs or DC or WH, equation (2) indicates the total annual transportation cost and handling cost to satisfy replenishment orders from retailers, equation (3) describes the total annual transportation cost, fixed ordering cost and handling cost to meet the replenishment orders of the hubs by other hubs or orders from DCs by WH, equation (4) the total annual transportation cost, fixed ordering cost and handling cost to meet the replenishment orders of the hubs by the sources or WH by the plant, equation (5) introduces the penalty cost for replenishment orders from retailers unmet immediately which is defined linear to the average demands and product value at retailers. Equation (6) - (9) describe the constraints that the replenishment orders can only be met by one facility each time. Hence, order splitting is not allowed in the model. Equation (10) indicates that the demanding quantity cannot be allowed fractional or partial.

# 4 Results analysis

## 4.1 Experiments Design

With optimal replenishment parameters at retailers obtained by (Giard 2005), the dynamic source selection strategy Source Substitution in (Pan et al. 2014) is applied to determine the source selection variables $x_{0in}/x_{kin}/\ x_{kjn}$ . Recall the Source Substitution strategy always chooses the nearest candidate to the ordering point among the candidates with inventory level bigger than the ordering quantity. A simulated annealing algorithm is constructed to optimize the replenishment parameters of intermediate stocking points. With the optimal replenishment parameters for each hub, we simulated the total system for 100 times and evaluate the average total cost of the hub system, the average total inventory level of all hubs, the average end customer level at retailers, and etc.

A single product network with a supplier company supplying two regional retailer companies is studied and the localization is shown in Figure 1. As seen in the picture, the three companies have common areas of logistic activities and geographically privileged areas. The points of sales R1 and R2 are geographically privileged to their own retailer company while the points of sales R3 and R4 are in the common area of the three companies. It indicates the three companies have great possibilities to share their logistic activities, i.e. transportation, storage capacity, and etc. The lead-time are assumed to be 5 days between the plant and the hubs, 1 day between hubs and 2 days between hubs and retailers. The route distance matrix (km) is described in the Table 1 as follows.

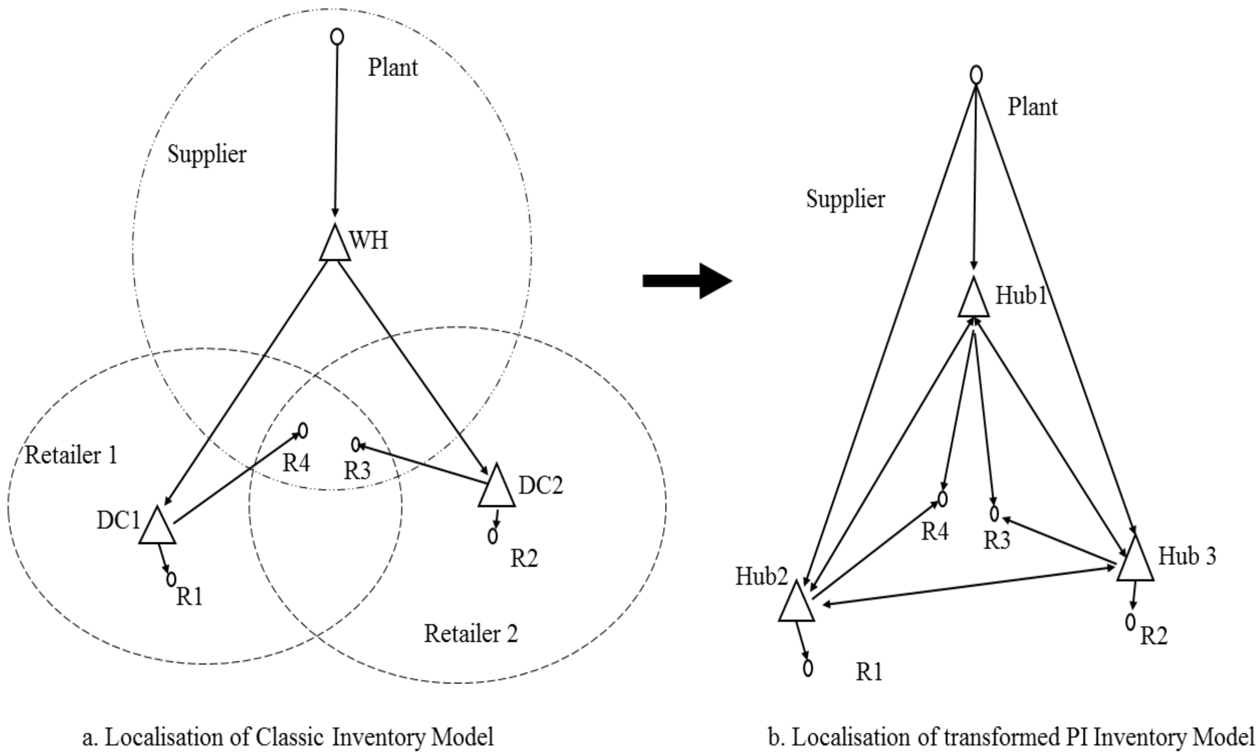



*Figure 1. Localization of the facilities in networks*

*Table 1. Route Distance Matrix (km)*

| Destination \ Source | Plant | HUB1 | HUB2 | HUB3 |
|---|---|---|---|---|
| HUB1 | 200 | 0 | 160 | 155 |
| HUB2 | 350 | 160 | 0 | 165 |
| HUB3 | 325 | 155 | 165 | 0 |
| R1 | | 170 | 15 | 160 |
| R2 | | 140 | 165 | 15 |
| R3 | | 100 | 95 | 90 |
| R4 | | 90 | 100 | 105 |

The impact of the following four parameters are analyzed: fixed ordering cost A= 8/20/80 (monetary unit) per order (or 0 if the order is not satisfied), the average demand level at retailers (value range: high/low, seen in Table 2), the product value 0.5/50 (monetary unit) leading to different penalty cost and holding cost, and the handling cost 0/2 (monetary unit) for a pallet of goods enter and out of each stocking point. For this paper, the low standard deviation at retailers are considered and the values are shown in Table 2. A pallet of goods is assumed to be 1.73 $m^3$ and a unit of goods is assumed to be 0.003 $m^3$ (equivalent to 500ml or 750 ml bottle of beverage). A SKU is a container of 6 units of goods. A full truck-load is assumed to be 33 full pallets. The penalty cost for retailers' orders unmet immediately is considered as 20% of the product value in intermediate stocking points and 30% at retailers. The transportation cost from the plant to hubs and among hubs is assumed to be 1.4 (monetary unit) per full truckload per km and 2.0 per full truckload per km from hubs to retailers. This assumption is based on the fact that the long-haul transportation cost is lower than that the last mile transportation cost. The holding cost per SKU is considered as the sum of two parts: the stocking cost per unit per day for storage space that is assumed to be 0.11 per $m^3$ /day in WH or DC or hubs (0.165 at retailers), and capital cost which is 8% of the product value per year. Therefore, for scenario of each network, there are 3*2*2*2=24 instances, as indexed in Table 3.

*Table 2. Daily demand level and standard deviation at retailers (units of goods)*

| Nodes | R1 | R2 | R3 | R4 |
|---|---|---|---|---|
| u1 (High) | 240 | 288 | 144 | 192 |
| u2 (low) | 30 | 45 | 20 | 46 |
| σ (Low) | 3 | 4 | 2 | 3 |

*Table 3 List of instances*

| Instances Index | Handling cost | Fixed ordering cost | Product value | Demand level |
|---|---|---|---|---|
| 1 | 0 | 8 | 0.5 | u1 |
| 2 | 0 | 8 | 0.5 | u2 |
| 3 | 0 | 8 | 50 | u1 |
| 4 | 0 | 8 | 50 | u2 |
| 5 | 0 | 20 | 0.5 | u1 |
| 6 | 0 | 20 | 0.5 | u2 |
| 7 | 0 | 20 | 50 | u1 |
| 8 | 0 | 20 | 50 | u2 |
| 9 | 0 | 80 | 0.5 | u1 |
| 10 | 0 | 80 | 0.5 | u2 |
| 11 | 0 | 80 | 50 | u1 |
| 12 | 0 | 80 | 50 | u2 |
| 13 | 2 | 8 | 0.5 | u1 |
| 14 | 2 | 8 | 0.5 | u2 |
| 15 | 2 | 8 | 50 | u1 |
| 16 | 2 | 8 | 50 | u2 |
| 17 | 2 | 20 | 0.5 | u1 |

| | | | | |
|---|---|---|---|---|
| 18 | 2 | 20 | 0.5 | u2 |
| 19 | 2 | 20 | 50 | u1 |
| 20 | 2 | 20 | 50 | u2 |
| 21 | 2 | 80 | 0.5 | u1 |
| 22 | 2 | 80 | 0.5 | u2 |
| 23 | 2 | 80 | 50 | u1 |
| 24 | 2 | 80 | 50 | u2 |

With all these setups and the methods presented in (Giard 2005), the optimal replenishment parameters (SKUs) for retailers are obtained to minimize its annual total cost including inventory holding cost, fixed ordering cost and penalty cost for lost sales with required lead time constraints. It is considered as the input data for the network of hubs, shown in Table 4. The optimal replenishment parameters for hubs are obtained by a simulated annealing heuristic method where the convergence ratio, the maximum temperature and the maximum iteration number are set to be 3%, 60, and 6000. With the optimal replenishment parameters for each hub, we simulated the total system for 100 times and evaluate the average total cost of the hub system, the average total inventory level of all hubs, the average end customer level at retailers, and etc. All the experimental tests are developed in Mathematica® 10.0 on a PC with Intel (R) Core (TM) i7-3940XM CPU 3.20 GHz and 32 Go RAM.

*Table 4. Optimal replenishment parameters of retailers (SKUs)*

| Instances Index | | R1 (q, s) SKUs | R2 (q, s) SKUs | R3 (q, s) SKUs | R4 (q, s) SKUs |
|---|---|---|---|---|---|
| 1 | 13 | {420, 121} | {326, 74} | {460, 145} | {376, 97} |
| 2 | 14 | {149, 16} | {183, 24} | {122, 11} | {184, 24} |
| 3 | 15 | {97, 122} | {75, 75} | {106, 145} | {87, 98} |
| 4 | 16 | {34, 17} | {42, 25} | {28, 11} | {42, 25} |
| 5 | 17 | {664, 121} | {515, 73} | {728, 145} | {594, 97} |
| 6 | 18 | {235, 16} | {288, 24} | {192, 10} | {291, 24} |
| 7 | 19 | {153, 122} | {119, 75} | {168, 145} | {137, 98} |
| 8 | 20 | {54, 17} | {67, 25} | {45, 11} | {67, 25} |
| 9 | 21 | {1329, 121} | {1029, 73} | {1456, 145} | {1188, 97} |
| 10 | 22 | {470, 15} | {576, 23} | {384, 7} | {582, 23} |
| 11 | 23 | {305, 122} | {237, 74} | {334, 145} | {273, 98} |
| 12 | 24 | {108, 17} | {132, 25} | {88, 11} | {134, 25} |

## 4.2 Results analysis

We adapt the performance ratio defined in (Ng et al., 2001) to compare the performance of scenarios. Here the classic inventory model is always used as the baseline and the performance ratio to other scenarios is the relative variations. Table 5 presents the average performance ratios of PI inventory model to classic inventory model with pre-determined sources. For example, for the instances where the average demand level is high as seen in Table 5, the average total cost of the distribution network are reduced 34% by the PI inventory model and 24% of the reduction comes from the holding cost while reaching a similar end customer service level at retailers. For all instances, we observe that the proposed PI inventory model can reduce the total logistic cost compared to classic inventory models while reaching a comparable or improved end customer service levels at retailers.

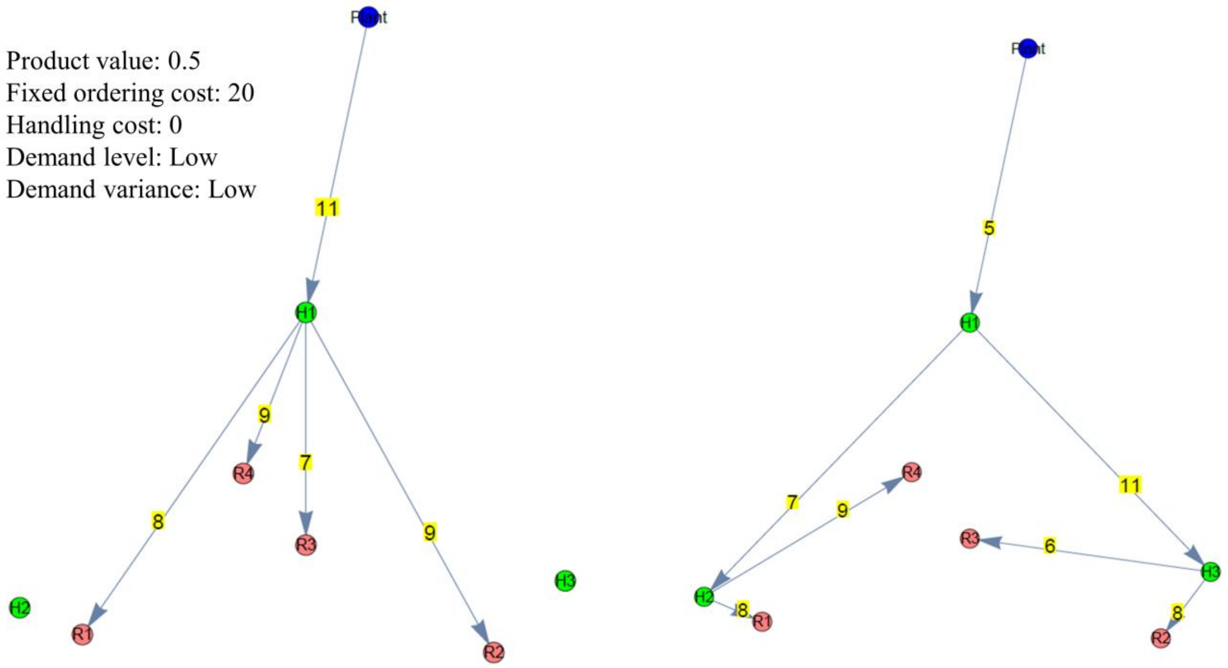
*Table 5. Average performance ratios: PI vs Classic*

| Parameter | Demand level | | Handling cost | | Product value | | Fixed ordering cost | | |
|---|---|---|---|---|---|---|---|---|---|
| Value | Low | High | 0 | 2 | 0.5 | 50 | 8 | 20 | 80 |
| Total cost | -54% | -34% | -43% | -45% | -44% | -44% | -39% | -39% | -53% |
| Service level | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| Holding | -48% | -24% | -38% | -34% | -36% | -36% | -29% | -30% | -50% |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Total Transportation | 1% | -4% | -2% | -1% | -2% | -1% | -4% | -1% | 1% |
| Downstream Transportation | 5% | 5% | 5% | 4% | 9% | 1% | 5% | 6% | 4% |
| Upstream Transportation | -4% | -9% | -7% | -5% | -10% | -2% | -9% | -7% | -2% |
| Penalty | -1% | 1% | 0% | 0% | 0% | 0% | 0% | -1% | 1% |
| Ordering | -5% | -3% | -4% | -4% | -2% | -6% | -3% | -5% | -3% |
| Handling | -2% | -4% | 0% | -5% | -4% | -2% | -3% | -3% | -2% |

In these instances compared to classic inventory model, the PI network has averagely 18% percent transportation cost profit perspective by direct shipping when the objective is to minimize the travel distance. However, as the objective is to minimize the annual total cost, the results shows direct shipping is not always the optimal solutions. And the transportation cost is not always reduced by 18%. We observe that in these scenarios the profits mainly come from the reduction of inventory holding cost and the average saving of the total cost reaches to 44% while reaching a similar end customer service level at retailers. Besides, the percentage of reduction of total cost increases with low demand level and high fixed ordering cost. Because with low level demand retailers at the network and high fixed ordering cost, the PI inventory model will compromise the downstream transportation cost from hubs to retailers to reduce the total inventory holding cost by grouping the distribution flows into one stocking point such as in Figure 2.(a). However, in the classic inventory model like in Figure 2.(b), the flow directions are always pre-determined and can't adapt to the changes of the economic environment changes.

*Figure 2. Distribution flow compare PI vs Classic*




a. PI inventory network

b. Classic inventory network

Concerning physical effects of the PI inventory model, we can see that different configurations of the four parameters restrict logistic activities of the PI network, resulting in different configurations of distribution flows. When the fixed ordering cost is low and the product value is high, the distribution flows are partially centralized and partially decentralized to minimize the total cost as seen in Figure 3. As the fixed ordering cost increases and the average demand level is low, the PI inventory model tends to group the distribution flows, an example as depicted in Figure 2.(a). Table 6 gives a quantitative description of interactions among hubs. The percentage of transshipment is defined as the percentage of replenishment orders of hubs satisfied by other hubs and the percentage of multi-sourcing refers to the percentage of retailer's orders satisfied by other hubs except the most regular hub. Results show that generally as the transportation cost set is linear to the quantity and distance travelled, the distribution flows of the network tend to be decentralized as to reduce the travel distances. However, the possibility of interactions among hubs increases when the product value and demand level at retailers increases as to reduce the total inventory holding cost and penalty cost for demands unmet immediately by pooling of the inventories among hubs. Besides, the augmenting fixed ordering cost and handling cost restrict the stock movements among hubs as expenses are charged for additional movements.

*Figure 3. Distribution flow PI network*

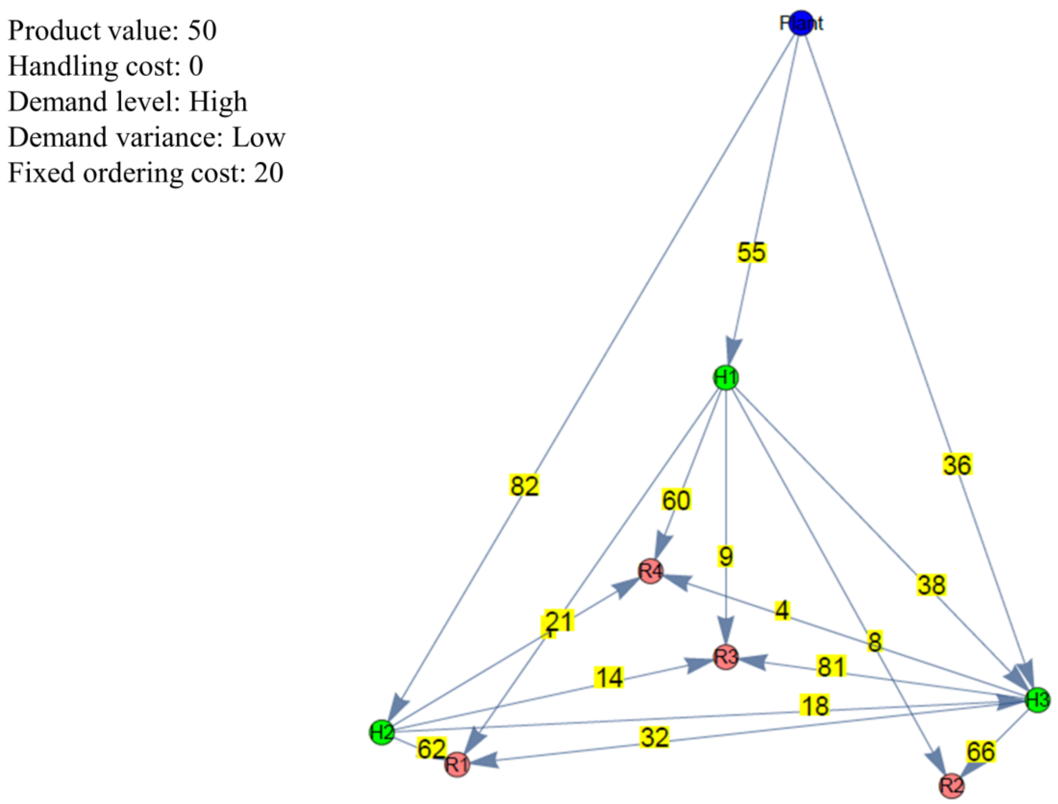



*Table 6 Interactions among hubs in PI network*

| Parameter | u | | hd | | c | | A | | |
|---|---|---|---|---|---|---|---|---|---|
| Value | Low | High | 0 | 2 | 0.5 | 50 | 8 | 20 | 80 |
| Percentage transshipment | 0% | 2% | 2% | 0% | 0% | 2% | 0% | 3% | 0% |
| Percentage multi-sourcing | 0% | 10% | 6% | 4% | 4% | 6% | 8% | 4% | 3% |

## 5 Conclusions

We have developed and evaluated a PI-inventory model in FMCG sector with a plant supplying a network of hubs and retailers. The optimal replenishment policies for hubs are determined to minimize the annual total cost of the distribution system including holding cost, fixed ordering cost, transportation cost and penalty cost for orders from retailers unmet immediately. A heuristic using simulated annealing is proposed to solve the optimization problem and then a simulation study is taken to evaluate the optimal policies under each sourcing strategy. The results show the PI inventory model can significantly reduce the average total cost compared to the current pre-determined inventory model while reaching a similar end customer service levels as the open and interconnected network enables more options. The profits are mainly come from the interactions among hubs to reduce the inventory levels around the network. Besides, as the source selection decisions are dynamically determined in PI network, a change in flexibility, i.e. fixed ordering cost, customer demand level, is immediately valued in the structure of the distribution flows, which improves the robustness of the inventory system to the perturbations of the economic environment. In a word, there is little doubt, this scheme proposes a drastic change compare to nowadays FMCG supply chain management that needs thus to be further investigated.

In this paper, the interactions among hubs are limited by the limited geography cover as well the small numbers of consignees in common logistic area. Great numbers of facilities in large common logistic area will lead to a significant change of the distribution scheme with more frequent transshipment among hubs and also more use of multi-sourcing in the downstream distribution flows. Further research is also required to investigate performance of different source selection methods as discussed in our previous paper (Pan et al. 2014) for different configurations of the network. Finally, another important factor to study is the robustness of PI network to some disruptive effect, for example the effects when a hub becomes suddenly unserviceable.

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