Constitutive modeling of a commercially pure titanium: validation using bulge tests
Benoit Revil-Baudard, Elisabeth Massoni

To cite this version:
Constitutive modeling of a commercially pure titanium: validation using bulge tests

Benoit Revil-Baudard¹ and Elisabeth Massoni²

¹Department of Mechanical and Aerospace Engineering, University of Florida, REEF, 1350 N. Poquito Rd., Shalimar, FL 32579, USA
²MINES ParisTech, Center for Material Forming (CEMEF), UMR CNRS 7635, BP 207, 06904 Sophia-Antipolis Cedex, France

E-mail: revil@ufl.edu

Abstract. In this paper, mechanical tests aimed at characterizing the plastic anisotropy of a commercially pure α-titanium sheet are presented. Hemispheric and elliptic bulge tests conducted to investigate the forming properties of the material are also reported. To model the particularities of the plastic response of the material the classical Hill [1] yield criterion, and Cazacu et al. [2] yield criterion are used. Identification of the material parameters involved in both criteria is based only on uniaxial test data, while their predictive capabilities are assessed through comparison with the bulge tests data. Both models reproduce qualitatively the experimental plastic strain distribution and the final thickness of the sheet. However, only Cazacu et al. [2] yield criterion, which accounts for both the anisotropy and tension-compression asymmetry of the material captures correctly plastic strain localization, in particular its directionality. Furthermore, it is shown that accounting for the strong tension-compression asymmetry in the model formulation improves numerical predictions regarding the mechanical behavior close to fracture of a commercially pure titanium alloy under sheet metal forming processes.

1. Introduction

Titanium and its alloys have outstanding mechanical properties such as moderate weight, high ductility and high strength, exceptional corrosion resistance, and excellent high-temperature properties. In particular, in the last decade considerable efforts have been devoted to the mechanical characterization of the response of commercially purity (CP) and high-purity (HP) titanium for quasi-static uniaxial loadings [3] [4]. It was reported that CP and HP titanium materials display strength differential effects, specifically in uniaxial compression the flow stress and strength are higher than in uniaxial tension. These strength differential effects are attributed mainly to mechanical twinning. Forming of titanium and its alloys still relies on numerous trial-and-error experiments. Given the complexity of forming processes and the cost of titanium, mathematical models for predicting the plastic strain distribution, strain localization, and failure are essential for realizing the true potential of these processes. However previous studies have also shown that classic plasticity models, such as J2 plasticity, or Hill [1] yield criterion cannot accurately describe the plastic behavior of titanium materials. The use of the von Mises yield criterion is clearly inadequate because it neglects the material’s plastic anisotropy. Although Hill [1]’s criterion accounts for anisotropy, it cannot predict accurately the tension-compression asymmetry of HP-titanium [3].
The main objective of this paper is to assess the capabilities of existing orthotropic models to capture the room-temperature plastic behavior of a commercially pure titanium for complex 3D loadings. The tension-compression asymmetry of the plastic flow is a key feature of the plastic behavior of HP and CP-Ti. Thus, Cazacu et al. [2] orthotropic criterion that accounts for yielding asymmetry between tension and compression associated with deformation twinning will be considered along with the classical Hill [1] criterion.

2. Constitutive modeling of pure Ti

The material used in this study is a CP $\alpha$-titanium sheet of 1.6 mm thickness, called T40. To quantify the plastic anisotropy in strains and in yield and flow stresses, uniaxial tests were carried out. Based on these tests, it can be concluded that the material displays a strong orthotropy in terms of yielding and Lankford coefficients (e.g. see fig.2). Also, the strength in uniaxial tension is lower than the strength in uniaxial compression for an in-plane orientations. The anisotropic properties of this material have been modeled by the classical Hill [1] yield criterion and by the Cazacu et al. [2] yield criterion, which also accounts for the observed tension-compression asymmetry. The anisotropic developed by Cazacu et al. [2] yield criterion is briefly given in what follows. The effective stress associated with the yield criterion is:

$$\bar{\sigma} = B \left[ \left( |\tilde{\sigma}_1| - k\tilde{\sigma}_1 \right)^2 + \left( |\tilde{\sigma}_2| - k\tilde{\sigma}_2 \right)^2 + \left( |\tilde{\sigma}_3| - k\tilde{\sigma}_3 \right)^2 \right]^\frac{1}{2} \quad (1)$$

where $k$ is an internal variable, its range of variation being (-1, 1), while $\tilde{\sigma}_1$, $\tilde{\sigma}_2$ and $\tilde{\sigma}_3$ are the principal values of the transformed stress tensor $\tilde{\sigma} = [L] : S$

with $S$ being the Cauchy stress deviator applied to the material. In Eq.(2), $L$ is a fourth-order orthotropic and symmetric tensor. Modeling the anisotropy by means of this 4th order symmetric and orthotropic tensor $L$, ensures that the material response is invariant under any orthogonal transformation belonging to the symmetry group of the material. In the coordinate system associated with the rolling (RD), transverse (TD) and normal direction (ND) and in Voigt notations, this tensor is represented by a 6x6 matrix given by

$$[L] = \begin{bmatrix}
L_{11} & L_{12} & L_{13} & 0 & 0 & 0 \\
L_{12} & L_{22} & L_{23} & 0 & 0 & 0 \\
L_{13} & L_{23} & L_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & L_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & L_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & L_{66}
\end{bmatrix} \quad (3)$$

In Eq.(1), $B$ is a constant defined such that the equivalent stress $\bar{\sigma}$ reduces to the tensile stress along RD. Thus, $B$ is expressed is expressed in terms of the anisotropy coefficients $L_{ij}$ with $i, j = 1...3$ and the material parameter $k$ as follows:

$$B = \frac{1}{\left[ (|T_1| - kT_1)^a + (|T_2| - kT_2)^a + (|T_3| - kT_3)^a \right]^\frac{1}{a}} ; T_1 = \left( \delta_{ij} - \frac{1}{3} \right) L_{ij} \quad (4)$$

The identification of the parameters involved in Eq.(1), namely $k$ and the components of the tensor $L$ is done based on the experimental data obtained in uniaxial tension (tensile strengths and Lankford coefficients) and in uniaxial compression tests [5], the parameter of the anisotropic form of Cazacu et al. [2] criterion (Eq. 1) has been identified. To capture the evolution of the tension-compression asymmetry and changes in plastic behavior with the plastic strain, all the
material parameters involved in the expression of the yield function are considered to evolve with accumulated plastic deformation. Therefore, the shape of the yield surface evolves from an elliptical shape (low tension-compression asymmetry) for small plastic strain toward a triangular shape for large plastic strain (see Fig. 1). For comparison purposes, the Hill [1] yield criterion will also be applied to model the plastic behavior of this material. Since the plastic behavior of the titanium alloy is strongly anisotropic, the Hill [1] couldn’t account correctly for both anisotropy in stresses and Lankford coefficients. Since the main goal is to assess the capabilities to correctly reproduce the plastic strain and evolution of the thickness during cold forming, only the Lankford coefficients are used in the identification procedure for Hill [1] criterion. Note that the Cazacu et al. (2006), with only one additional parameters than Hill [1] also accounts for the tension-compression asymmetry. Figure 2 compares the respective evolution of the Lankford coefficients with the angle with respect to the rolling direction for an equivalent plastic strain of 0.1. Both yield criteria capture well the in-plane strain anisotropy of the titanium material.

3. Assessment of the predictive capabilities for cold forming

Experimental bulging tests coupled with digital image correlation have been performed and revealed specific strain localization zones [4]. The results of such tests will also allow full 3-D validation of the constitutive models. Identified based on uniaxial tests, the predictive capabilities of the Hill [1] and Cazacu et al. [2] anisotropic yield criteria are assessed on experimental hemispheric and elliptical bulge tests. Both yield criteria provide good prediction for the overall equivalent plastic strain isocontours and final thickness of the metal sheet. The Hill [1] lacks accuracy in describing the plastic behavior for plastic strain close to fracture strain and cannot predict the position and orientation of the strain localization zone (see fig. 3). Only the Cazacu et al. [2] yield criterion, which accounts for the tension-compression asymmetry of the plastic behavior captures correctly the plastic strain localization and its orientation. Furthermore, the evolution of the thickness with pressure confirms that the Cazacu et al. [2] criterion better predict the plastic behavior close to fracture. To complementary illustrate these differences; in the figure 4 is plotted the evolution of the thickness at the center of the metal sheet with the pressure of the injected water for both Hill [1] and Cazacu et al. [2] yield criteria. Until the pressure reaches 21 MPa, both anisotropic yield criteria predict a similar evolution for the thickness at the center of the metal sheet. But, the Cazacu et al. [2] yield criterion is able to capture the change in the evolution rate of the thickness leading to strain localization. It is worth noting that the difference between the thicknesses predicted by the two anisotropic function increases as the tension-compression asymmetry becomes larger. Thus, for metal sheet

![Figure 1. Yield loci for a CP α-titanium according the Cazacu et al [2] yield criterion for different level of accumulated plastic strain.](image1)

![Figure 2. Evolution of the Lankford coefficient according the Hill [1] yield criterion and the Cazacu et al [2] yield function.](image2)
forming, accounting of the tension-compression asymmetry of the plastic behavior provides a better description of the mechanical response of the material. By comparing the yield loci for the Hill [1] and the Cazacu et al. (2006) criteria and conclusions drawn from the bulge simulations, it is worth noting that the strain localization appears when the tension-compression asymmetry becomes significant.

Figure 3. Isocontours of equivalent plastic strain for an elliptic bulge test of CP titanium with the RD oriented along the major axis of the ellipse: (a) DIC measure, b) the Cazacu et al [2] yield criterion, (c) Hill [1] yield criterion.

Figure 4. Evolution of the thickness of the CP titanium sheet located at the top of the bulge with pressure for an elliptic bulge test (the RD is oriented along the major axis of the ellipse): comparison between the response given by the Hill [1] yield criterion and from the Cazacu et al [2] yield function.

4. Conclusions
The prediction of the strain localization zone is directly correlated to the tension-compression asymmetry effect in three dimensional loadings, and only an anisotropic yield criterion accounting for this key feature could correctly reproduce the plastic behavior close to fracture for an CP α-titanium alloy. Use of a particular yield criterion to predict the plastic behavior of a titanium alloy during cold forming processes greatly relies on the experimental data available to identify this model and on the predictive capabilities of the model needed. If the model must reproduce the overall mechanical behavior for small to medium plastic strain, the Hill [1] yield criterion is a reliable model. Otherwise, if the plastic behavior close to rupture must be accurately described and the prediction of the strain localization zone is necessary, the Cazacu et al. [2] presents a better option.

References