Convex Relaxation for Water Distribution Systems

28th European Conference on Operation Research

Gratien Bonvin, Sophie Demassey

Poznan, 05.07.16

Water urban cycle, drinking water distribution sytems and electricity consumption

Electricity consumption for Water Supply & Treatment in USA (EPRI 2002)

- Water supply : $\sim$ 73 TWh (1.7%)
- Wastewater treatment : $\sim$ 76 TWh (1.8%)

We focus on drinking water networks

- Electricity consumption : $\sim$ 35 TWh (0.8%)
- Electricity consumed by pumping stations

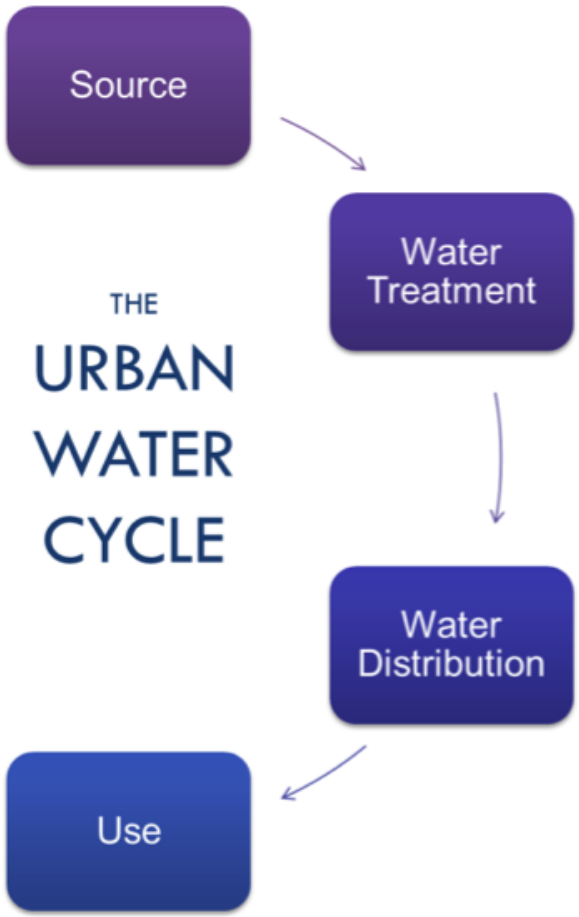
Water urban cycle, drinking water distribution sytems and electricity consumption

Electricity consumption for Water Supply & Treatment in USA (EPRI 2002)

- Water supply : $\sim$ 73 TWh (1.7%)
- Wastewater treatment : $\sim$ 76 TWh (1.8%)

We focus on drinking water networks

- Electricity consumption : $\sim$ 35 TWh (0.8%)
- Electricity consumed by pumping stations



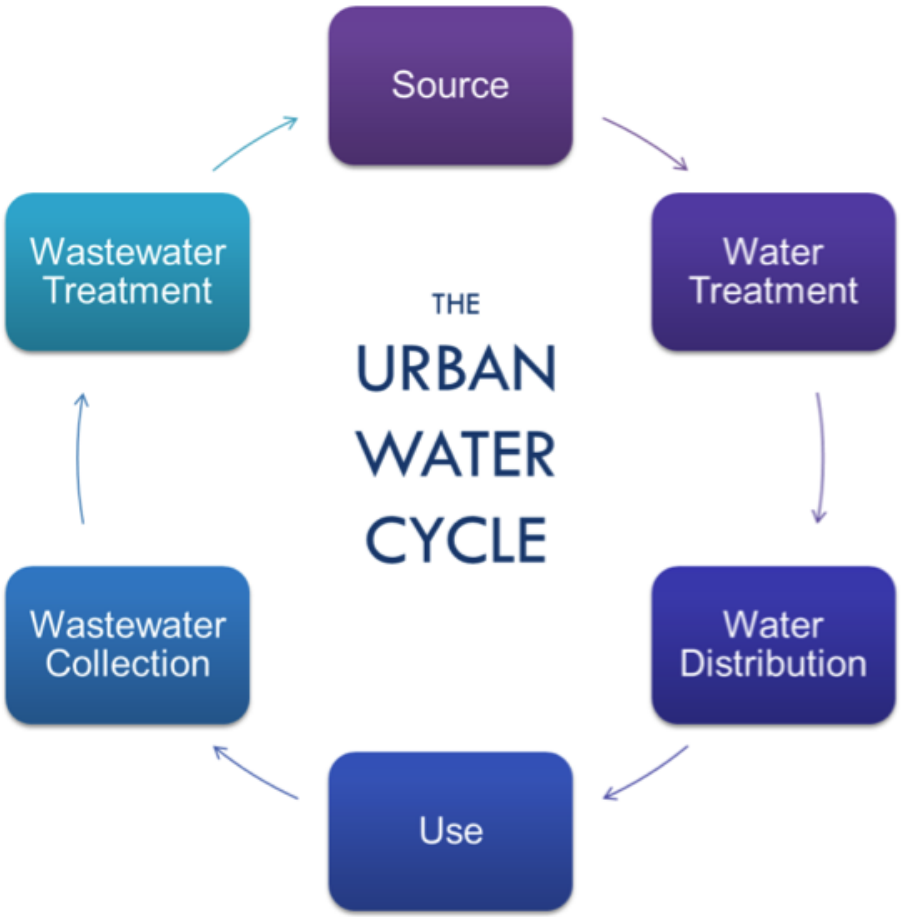
Water urban cycle, drinking water distribution sytems and electricity consumption

Electricity consumption for Water Supply & Treatment in USA (EPRI 2002)

- Water supply : $\sim$ 73 TWh (1.7%)
- Wastewater treatment : $\sim$ 76 TWh (1.8%)

We focus on drinking water networks

- Electricity consumption : $\sim$ 35 TWh (0.8%)
- Electricity consumed by pumping stations



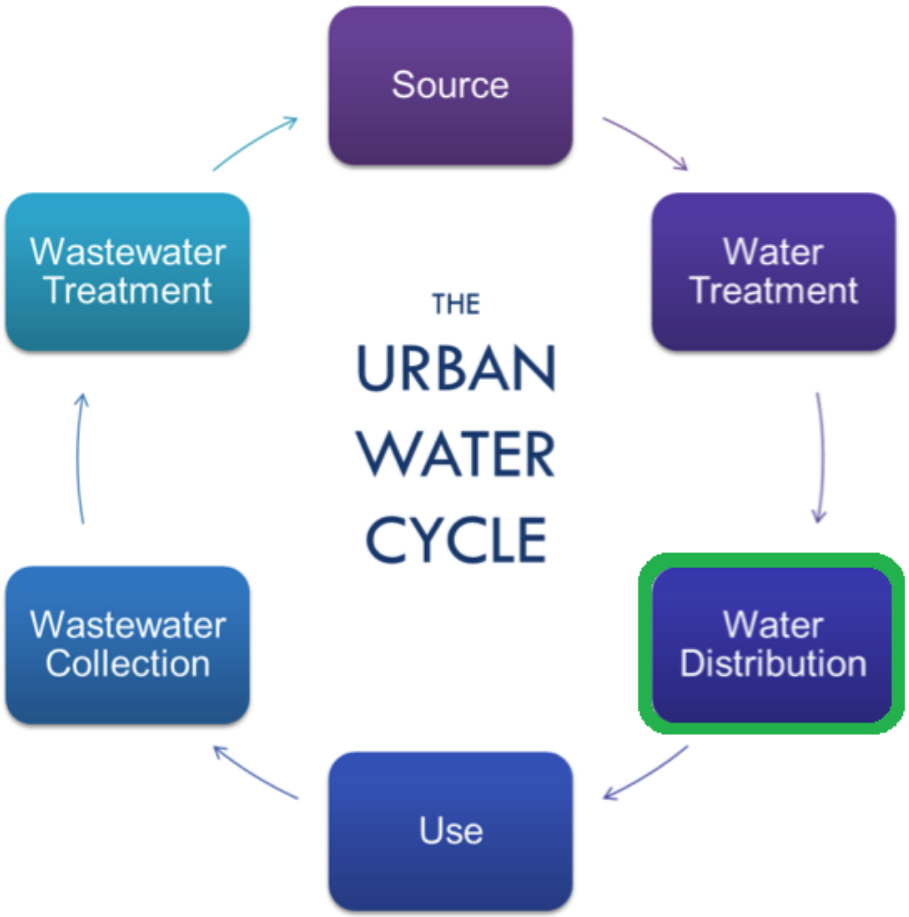
Water urban cycle, drinking water distribution sytems and electricity consumption

Electricity consumption for Water Supply & Treatment in USA (EPRI 2002)

- Water supply :
  $\sim$ 73 TWh (1.7%)
- Wastewater treatment :
  $\sim$ 76 TWh (1.8%)

We focus on drinking water networks

- Electricity consumption :
  $\sim$ 35 TWh (0.8%)
- Electricity consumed by pumping stations



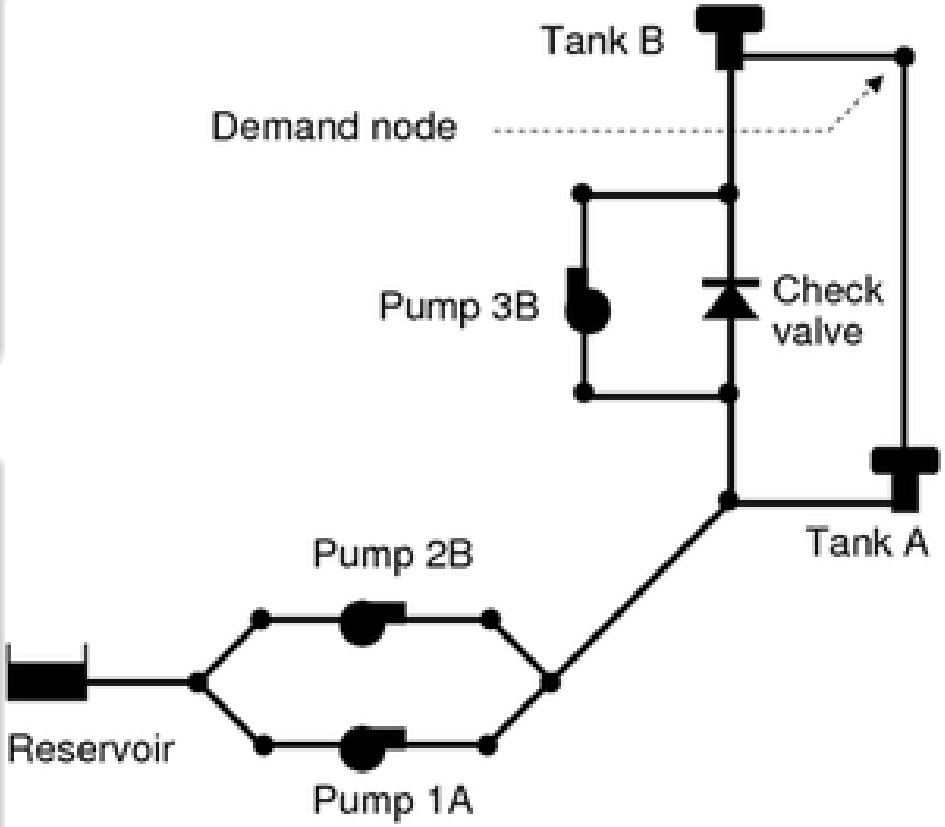
Water urban cycle, drinking water distribution sytems and electricity consumption

Electricity consumption for Water Supply & Treatment in USA (EPRI 2002)

- Water supply :
  $\sim$ 73 TWh (1.7%)
- Wastewater treatment :
  $\sim$ 76 TWh (1.8%)

We focus on drinking water networks

- Electricity consumption :
  $\sim$ 35 TWh (0.8%)
- Electricity consumed by pumping stations



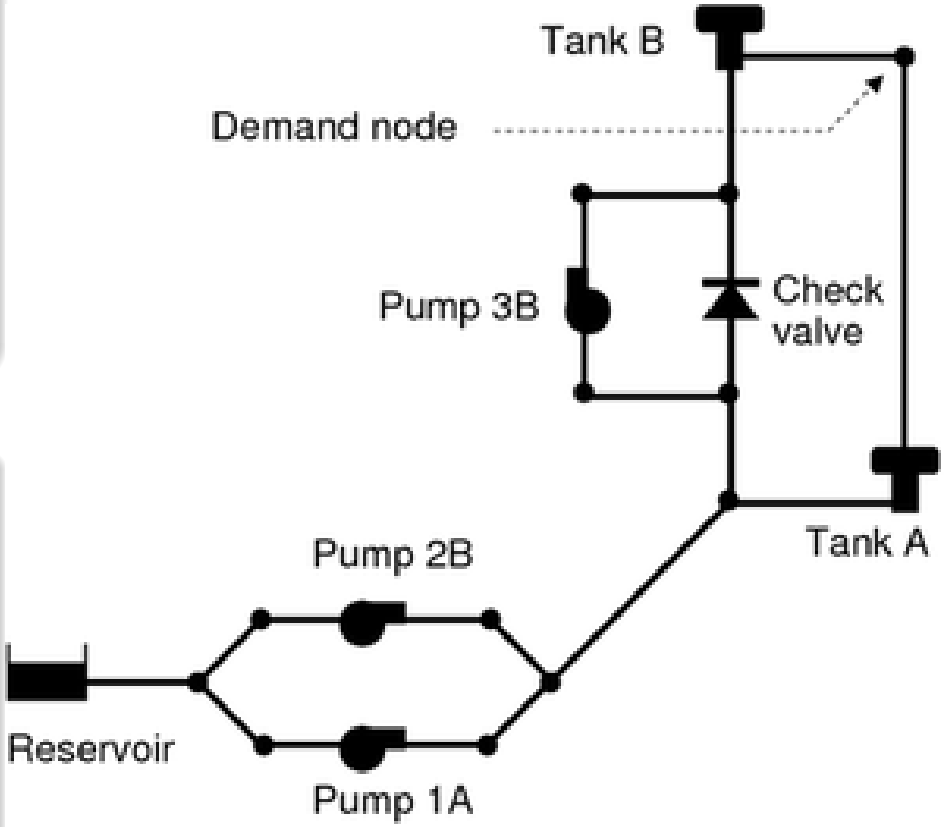
Water urban cycle, drinking water distribution sytems and electricity consumption

Electricity consumption for Water Supply & Treatment in USA (EPRI 2002)

- Water supply :
  $\sim$ 73 TWh (1.7%)
- Wastewater treatment :
  $\sim$ 76 TWh (1.8%)

We focus on drinking water networks

- Electricity consumption :
  $\sim$ 35 TWh (0.8%)
- Electricity consumed by pumping stations



How to minimize electrical costs due to pumping stations ?

Pumping stations and electricity markets

Current strategy : a step-by-step strategy driven by water volume thresholds

- Reservoir are filled to their maximal capacity during the night (off-peak tariffs)
- Reservoir are kept to their lower capacity during the day (peak tariff)

New challenges for the energy system and new opportunities for WDS

- Daily pump scheduling with day-ahed prices and electricity purchase on spot market (Ghaddar 2015)
- Increase the value of storage capacities with demand-response strategies (Menke 2016)

Pumping stations and electricity markets

Current strategy : a step-by-step strategy driven by water volume thresholds

- Reservoir are filled to their maximal capacity during the night (off-peak tariffs)
- Reservoir are kept to their lower capacity during the day (peak tariff)

New challenges for the energy system and new opportunities for WDS

- Daily pump scheduling with day-ahed prices and electricity purchase on spot market (Ghaddar 2015)
- Increase the value of storage capacities with demand-response strategies (Menke 2016)

Pumping scheduling : physical model

Flow equilibrium in nodes

$$\sum_{(i,j)} q_{ijt} - \sum_{(j,k)} q_{jkt} = D_{jt}$$

Flow equilibrium in reservoirs

$$V_{i(t+1)} - V_{it} = \left(\sum_{(j,i)} q_{ijt} - \sum_{(i,j)} q_{ijt} - D_{it}\right)\Delta_t$$

Head-flow relations for pipes and pumps

$$h_{it} - h_{jt} = \kappa_{ij} sign(q_{ijt}) q_{ijt}^{\gamma_{ij}}$$

$$h_{jt} - h_{it} = \Delta H_{ij} - \gamma_{ij}^{1} q_{ijt}^{\gamma_{ij}^{2}} \text{ si } x_{ijt} = 1$$

Minimize electrical costs

$$Cost = \sum_{(i,j)\in P, t\in T} \Gamma(q_{ijt}, x_{ijt}) C_t \Delta_t$$

$$\Gamma(q_{ijt}, x_{ijt}) = A x_{ijt} + B q_{ijt}$$

Pumping scheduling : physical model

Flow equilibrium in nodes

$$\sum_{(i,j)} q_{ijt} - \sum_{(j,k)} q_{jkt} = D_{jt}$$

Flow equilibrium in reservoirs

$$V_{i(t+1)} - V_{it} = \left(\sum_{(j,i)} q_{ijt} - \sum_{(i,j)} q_{ijt} - D_{it}\right)\Delta_t$$

Head-flow relations for pipes and pumps

$$h_{it} - h_{jt} = \kappa_{ij} sign(q_{ijt}) q_{ijt}^{\gamma_{ij}}$$

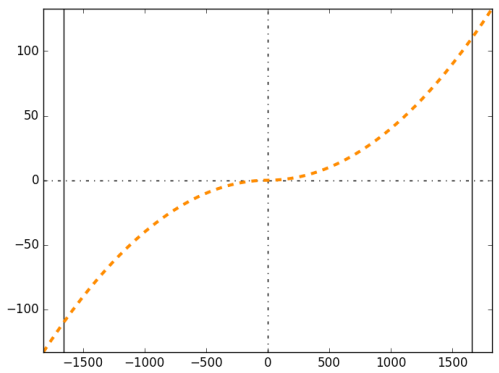
$$h_{jt} - h_{it} = \Delta H_{ij} - \gamma_{ij}^{1} q_{ijt}^{\gamma_{ij}^{2}} \text{ si } x_{ijt} = 1$$

Minimize electrical costs

$$Cost = \sum_{(i,j)\in P, t\in T} \Gamma(q_{ijt}, x_{ijt}) C_t \Delta_t$$

$$\Gamma(q_{ijt}, x_{ijt}) = A x_{ijt} + B q_{ijt}$$

Pumping scheduling : physical model



Flow equilibrium in nodes

$$\sum_{(i,j)} q_{ijt} - \sum_{(j,k)} q_{jkt} = D_{jt}$$

Flow equilibrium in reservoirs

$$V_{i(t+1)} - V_{it} = \left(\sum_{(j,i)} q_{ijt} - \sum_{(i,j)} q_{ijt} - D_{it}\right)\Delta_t$$

Head-flow relations for pipes and pumps

$$h_{it} - h_{jt} = \kappa_{ij} sign(q_{ijt}) q_{ijt}^{\gamma_{ij}}$$

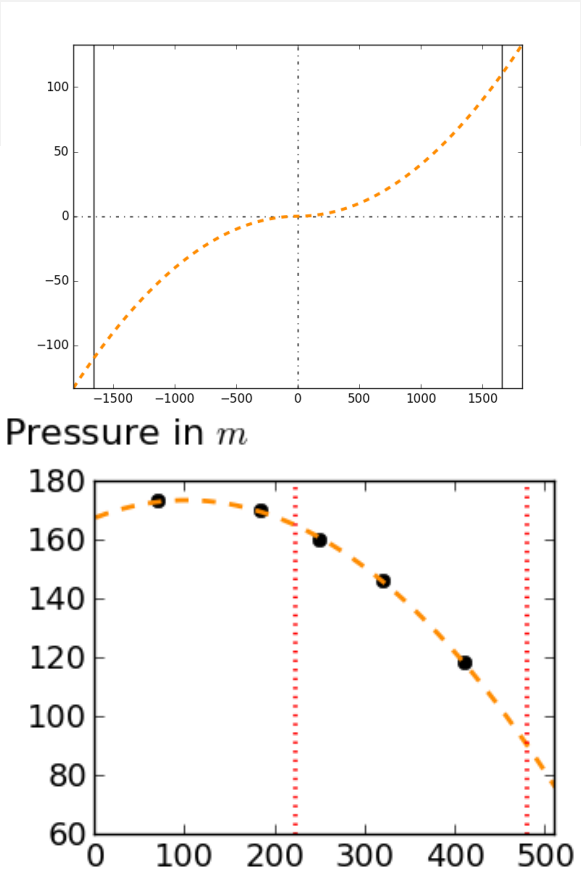
$$h_{jt} - h_{it} = \Delta H_{ij} - \gamma_{ij}^{1} q_{ijt}^{\gamma_{ij}^{2}} \text{ si } x_{ijt} = 1$$

Minimize electrical costs

$$Cost = \sum_{(i,j)\in P, t\in T} \Gamma(q_{ijt}, x_{ijt}) C_t \Delta_t$$

$$\Gamma(q_{ijt}, x_{ijt}) = A x_{ijt} + B q_{ijt}$$

Pumping scheduling : physical model



Flow equilibrium in nodes

$$\sum_{(i,j)} q_{ijt} - \sum_{(j,k)} q_{jkt} = D_{jt}$$

Flow equilibrium in reservoirs

$$V_{i(t+1)} - V_{it} = \left(\sum_{(j,i)} q_{ijt} - \sum_{(i,j)} q_{ijt} - D_{it}\right)\Delta_t$$

Head-flow relations for pipes and pumps

$$h_{it} - h_{jt} = \kappa_{ij} sign(q_{ijt}) q_{ijt}^{\gamma_{ij}}$$

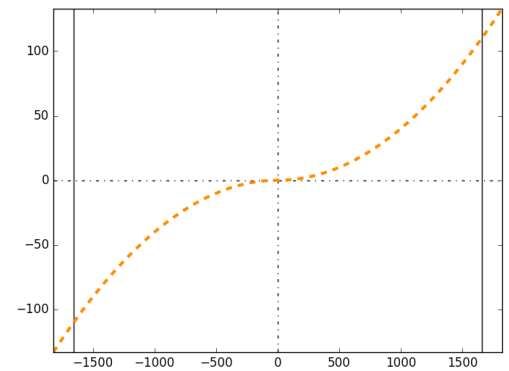
$$h_{jt} - h_{it} = \Delta H_{ij} - \gamma_{ij}^1 q_{ijt}^{\gamma_{ij}^2} \text{ si } x_{ijt} = 1$$

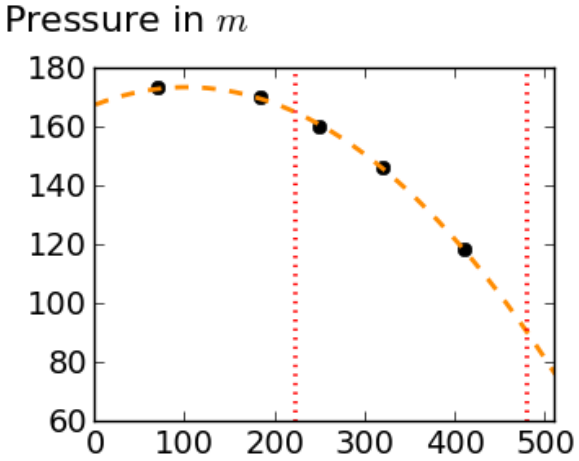
Minimize electrical costs

$$Cost = \sum_{(i,j)\in P, t\in T} \Gamma(q_{ijt}, x_{ijt}) C_t \Delta_t$$

$$\Gamma(q_{ijt}, x_{ijt}) = A x_{ijt} + B q_{ijt}$$

Pumping scheduling : physical model





Flow equilibrium in nodes

$$\sum_{(i,j)} q_{ijt} - \sum_{(j,k)} q_{jkt} = D_{jt}$$

Flow equilibrium in reservoirs

$$V_{i(t+1)} - V_{it} = \left(\sum_{(j,i)} q_{ijt} - \sum_{(i,j)} q_{ijt} - D_{it}\right)\Delta_t$$

Head-flow relations for pipes and pumps

$$h_{it} - h_{jt} = \kappa_{ij} sign(q_{ijt}) q_{ijt}^{\gamma_{ij}}$$

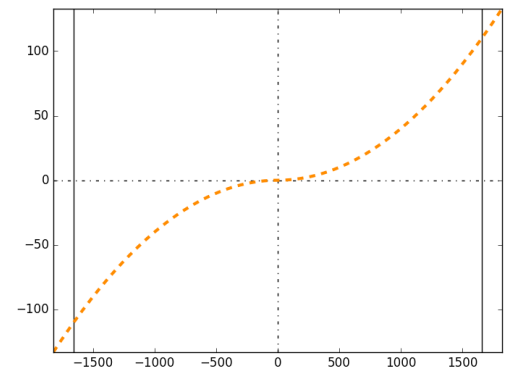
$$h_{jt} - h_{it} = \Delta H_{ij} - \gamma_{ij}^1 q_{ijt}^{\gamma_{ij}^2} \text{ si } x_{ijt} = 1$$

Minimize electrical costs

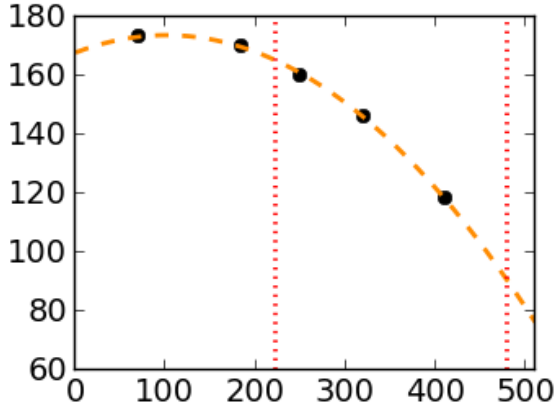
$$Cost = \sum_{(i,j)\in P, t\in T} \Gamma(q_{ijt}, x_{ijt}) C_t \Delta_t$$

$$\Gamma(q_{ijt}, x_{ijt}) = A x_{ijt} + B q_{ijt}$$

Pumping scheduling : physical model



Pressure in m



Flow equilibrium in nodes

$$\sum_{(i,j)} q_{ijt} - \sum_{(j,k)} q_{jkt} = D_{jt}$$

Flow equilibrium in reservoirs

$$V_{i(t+1)} - V_{it} = \left(\sum_{(j,i)} q_{ijt} - \sum_{(i,j)} q_{ijt} - D_{it}\right)\Delta_t$$

Head-flow relations for pipes and pumps

$$h_{it} - h_{jt} = \kappa_{ij} sign(q_{ijt}) q_{ijt}^{\gamma_{ij}}$$

$$h_{jt} - h_{it} = \Delta H_{ij} - \gamma_{ij}^1 q_{ijt}^{\gamma_{ij}^2} \text{ si } x_{ijt} = 1$$

Minimize electrical costs

$$Cost = \sum_{(i,j)\in P, t\in T} \Gamma(q_{ijt}, x_{ijt}) C_t \Delta_t$$

$$\Gamma(q_{ijt}, x_{ijt}) = A x_{ijt} + B q_{ijt}$$

Pumping scheduling : physical model

Flow equilibrium in nodes

$$\sum_{(i,j)} q_{ijt} - \sum_{(j,k)} q_{jkt} = D_{jt}$$

Flow equilibrium in reservoirs

$$V_{i(t+1)} - V_{it} = \left(\sum_{(j,i)} q_{ijt} - \sum_{(i,j)} q_{ijt} - D_{it}\right)\Delta_t$$

Head-flow relations for pipes and pumps

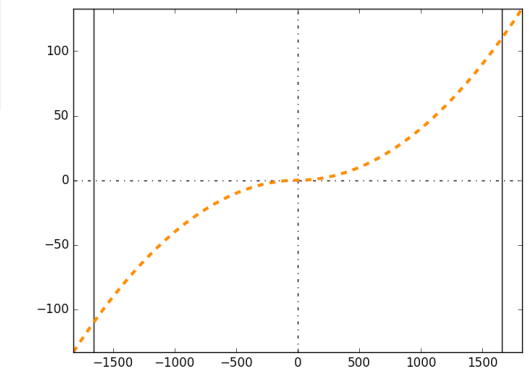
$$h_{it} - h_{jt} = \kappa_{ij} sign(q_{ijt}) q_{ijt}^{\gamma_{ij}}$$

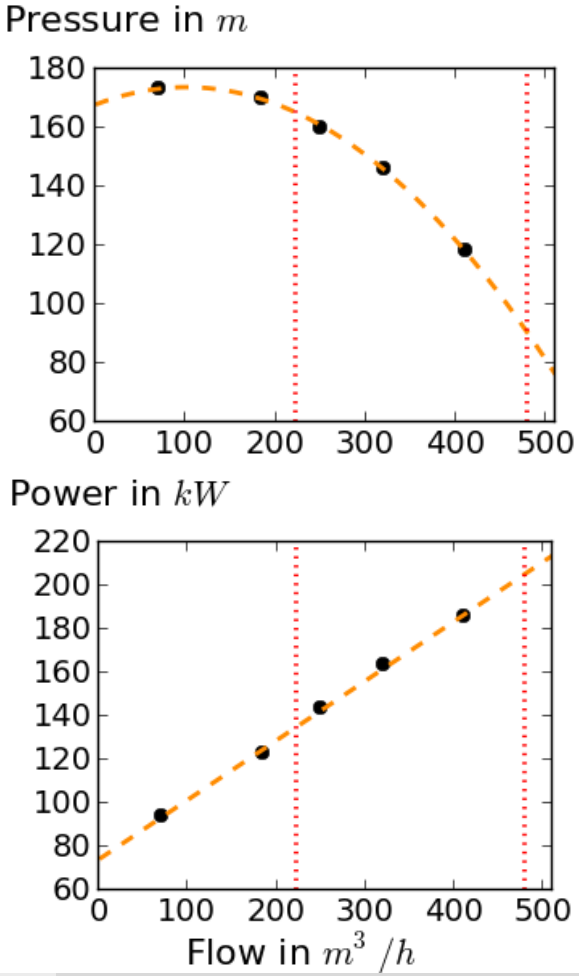
$$h_{jt} - h_{it} = \Delta H_{ij} - \gamma_{ij}^1 q_{ijt}^{\gamma_{ij}^2} \text{ si } x_{ijt} = 1$$

Minimize electrical costs

$$Cost = \sum_{(i,j)\in P, t\in T} \Gamma(q_{ijt}, x_{ijt}) C_t \Delta_t$$

$$\Gamma(q_{ijt}, x_{ijt}) = A x_{ijt} + B q_{ijt}$$





Why a convex relaxation ? (1/2)

Mathematical issues

- Non-convex constraints for head-flow relations
- Binary variables for pump status

State-of-the-art methods

- Piecewise linear approximation

  (Morsi 2012, Geissler 2013, Verleye 2013/2015)
- Non-convex optimisation

  (Burgschweiger 2009, Ghaddar 2015)

Our proposition

- $h_{jt} - h_{it} = \Delta H_{ij} - \gamma_{ij}^{1} q_{ijt}^{\gamma_{ij}^{2}}$ if $x_{ijt} = 1$
- $h_{it} - h_{jt} = \kappa_{ij} sign(q_{ijt}) q_{ijt}^{\gamma_{ij}}$

Why a convex relaxation ? (1/2)

Mathematical issues

- Non-convex constraints for head-flow relations
- Binary variables for pump status

State-of-the-art methods

- Piecewise linear approximation

  (Morsi 2012, Geissler 2013, Verleye 2013/2015)
- Non-convex optimisation

  (Burgschweiger 2009, Ghaddar 2015)

Our proposition

- $h_{jt} - h_{it} = \Delta H_{ij} - \gamma_{ij}^{1} q_{ijt}^{\gamma_{ij}^{2}}$ if $x_{ijt} = 1$
- $h_{it} - h_{jt} = \kappa_{ij} sign(q_{ijt}) q_{ijt}^{\gamma_{ij}}$

Why a convex relaxation ? (1/2)

Mathematical issues

- Non-convex constraints for head-flow relations
- Binary variables for pump status

State-of-the-art methods

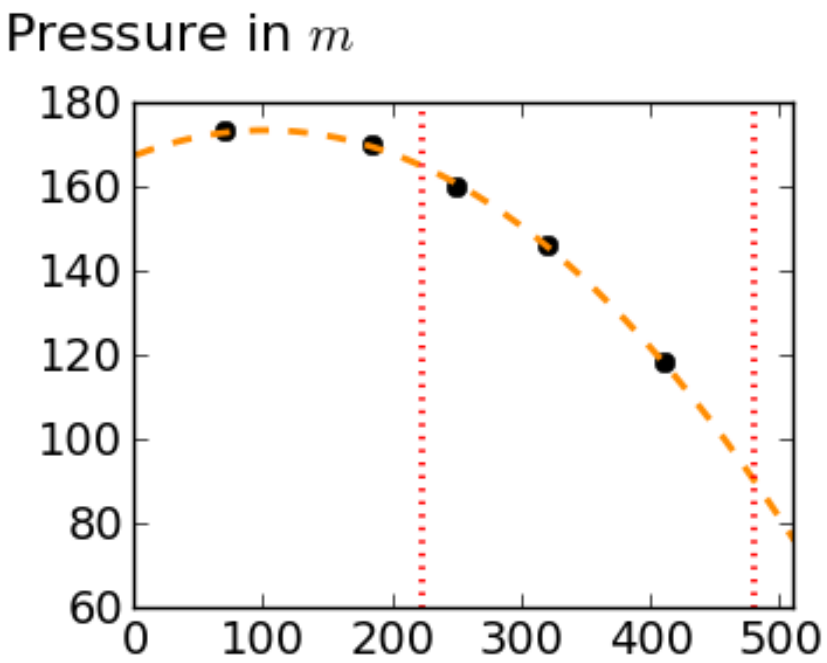
- Piecewise linear approximation

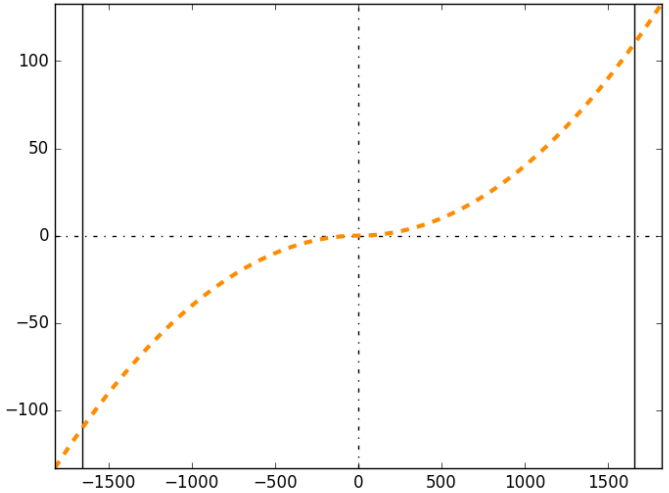
  (Morsi 2012, Geissler 2013, Verleye 2013/2015)
- Non-convex optimisation

  (Burgschweiger 2009, Ghaddar 2015)

Our proposition

- $h_{jt} - h_{it} = \Delta H_{ij} - \gamma_{ij}^1 q_{ijt}^{\gamma_{ij}^2}$ if $x_{ijt} = 1$
- $h_{it} - h_{jt} = \kappa_{ij} sign(q_{ijt}) q_{ijt}^{\gamma_{ij}}$





Why a convex relaxation ? (1/2)

Mathematical issues

- Non-convex constraints for head-flow relations
- Binary variables for pump status

State-of-the-art methods

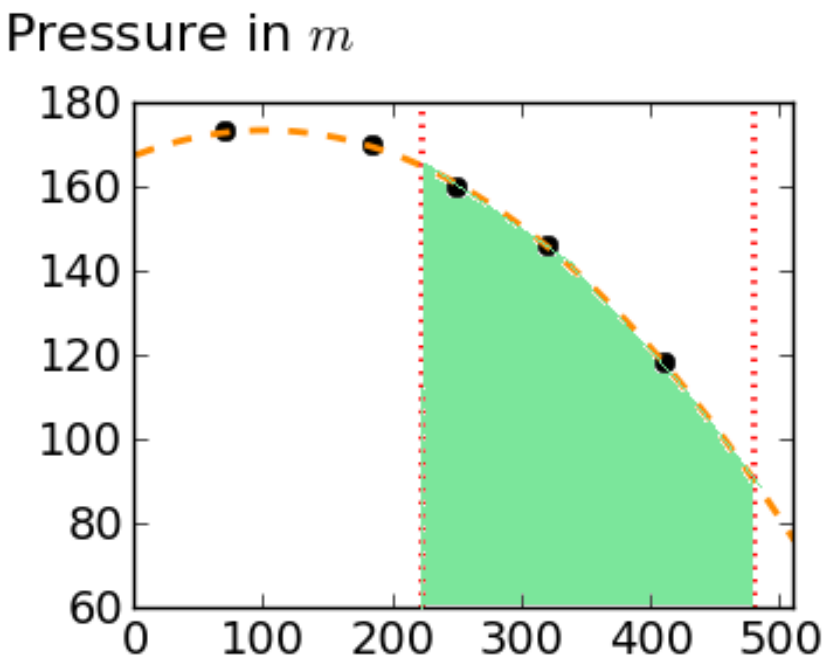
- Piecewise linear approximation

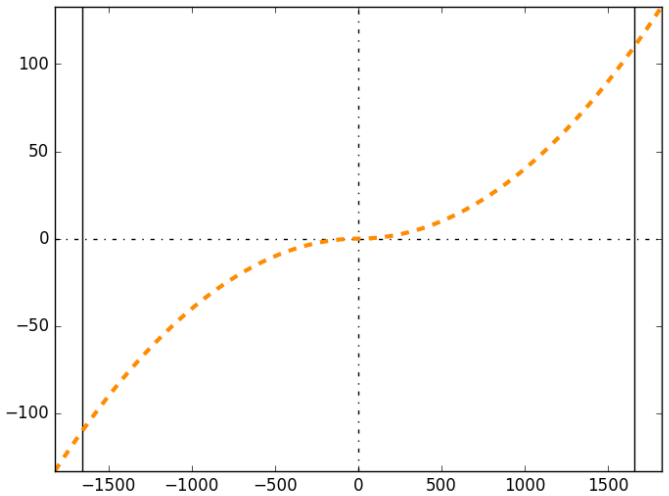
  (Morsi 2012, Geissler 2013, Verleye 2013/2015)
- Non-convex optimisation

  (Burgschweiger 2009, Ghaddar 2015)

Our proposition

- $h_{jt} - h_{it} \leq \Delta H_{ij} - \gamma_{ij}^{1} q_{ijt}^{\gamma_{ij}^{2}}$
- $h_{it} - h_{jt} = \kappa_{ij} sign(q_{ijt}) q_{ijt}^{\gamma_{ij}}$





Why a convex relaxation ? (1/2)

Mathematical issues

- Non-convex constraints for head-flow relations
- Binary variables for pump status

State-of-the-art methods

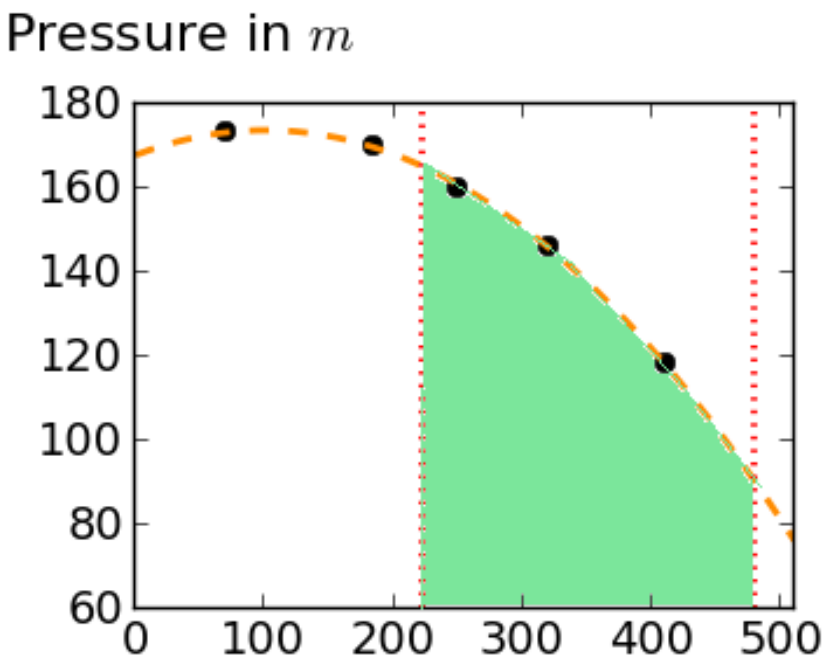
- Piecewise linear approximation

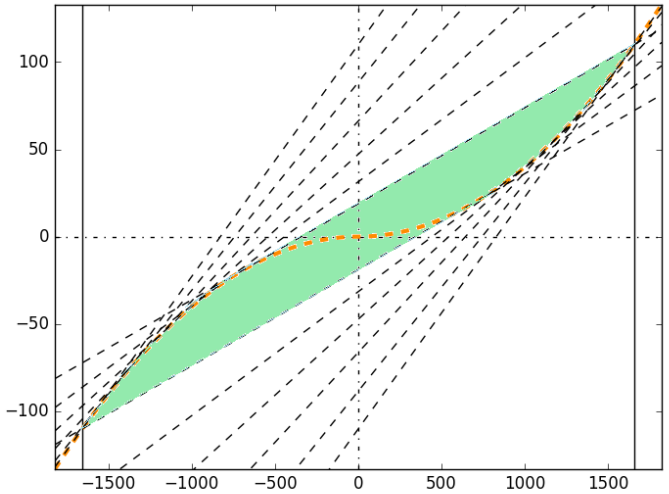
  (Morsi 2012, Geissler 2013, Verleye 2013/2015)
- Non-convex optimisation

  (Burgschweiger 2009, Ghaddar 2015)

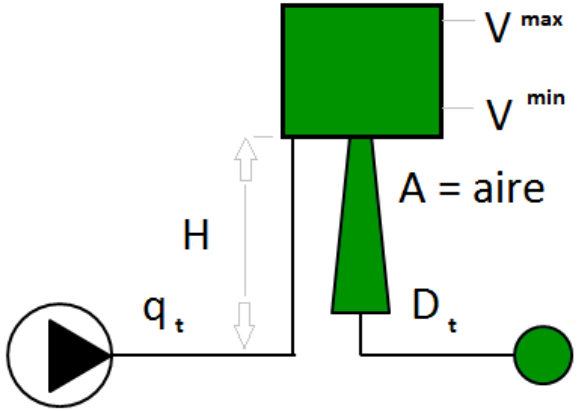
Our proposition

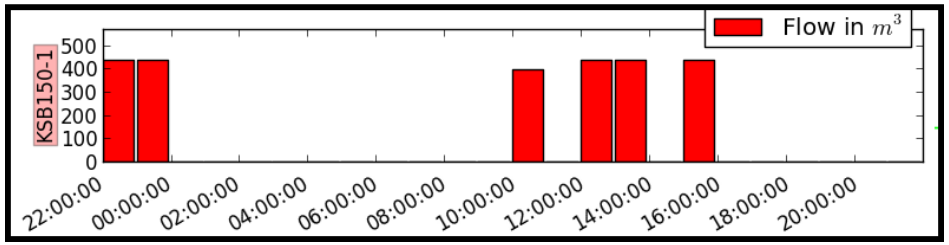
- $h_{jt} - h_{it} \leq \Delta H_{ij} - \gamma_{ij}^{1} q_{ijt}^{\gamma_{ij}^{2}}$
- $h_{it} - h_{jt} \in Conv\left(\kappa_{ij} sign(q_{ijt}) q_{ijt}^{\gamma_{ij}}\right)$

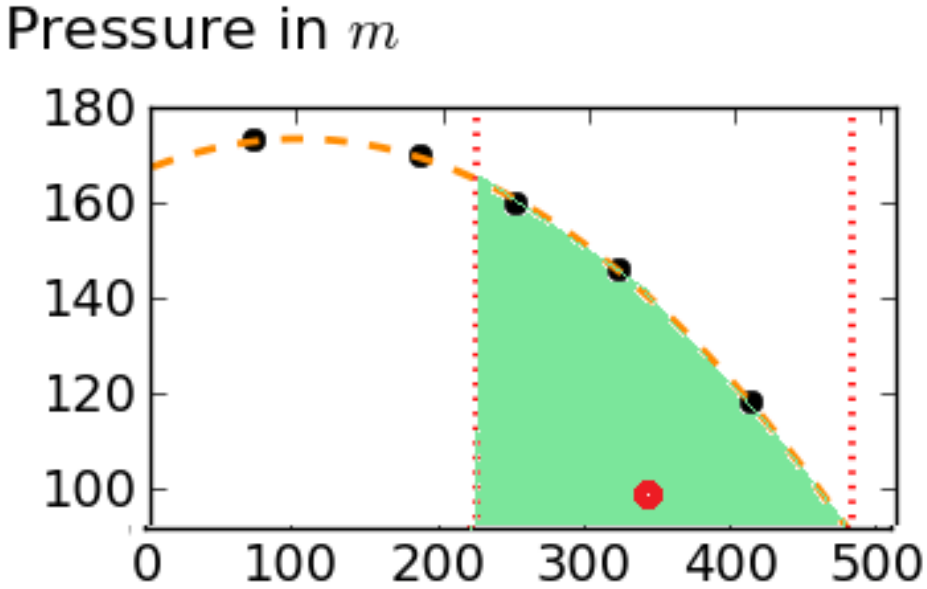


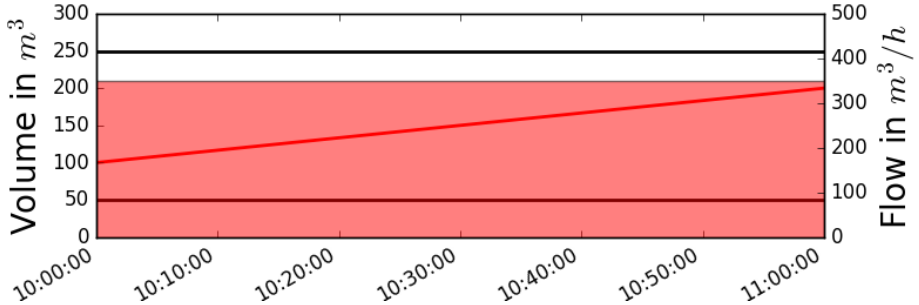


Why a convex relaxation ? (2/2)









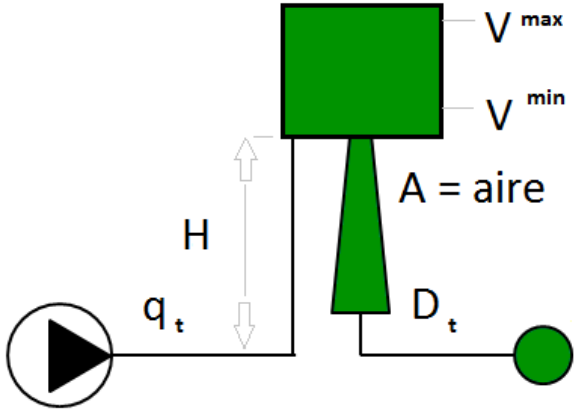
From a relaxed solution...

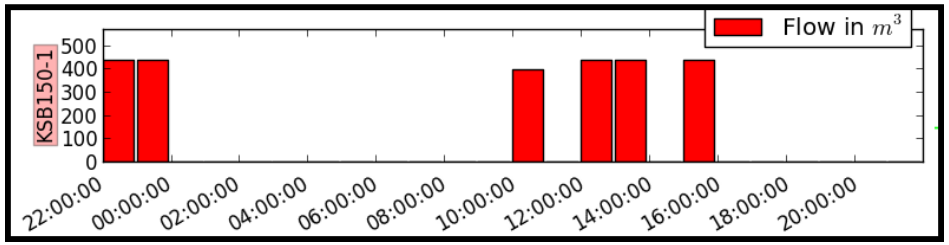
$$h_{jt} - h_{it} \leq \Delta H_{ij} - \gamma^1_{ij} q_{ijt}^{\gamma^2_{ij}}$$
$$Cost = \sum_{(i,j)\in P, t\in T} \Gamma(q_{ijt}, x_{ijt}) C_t \Delta_t$$

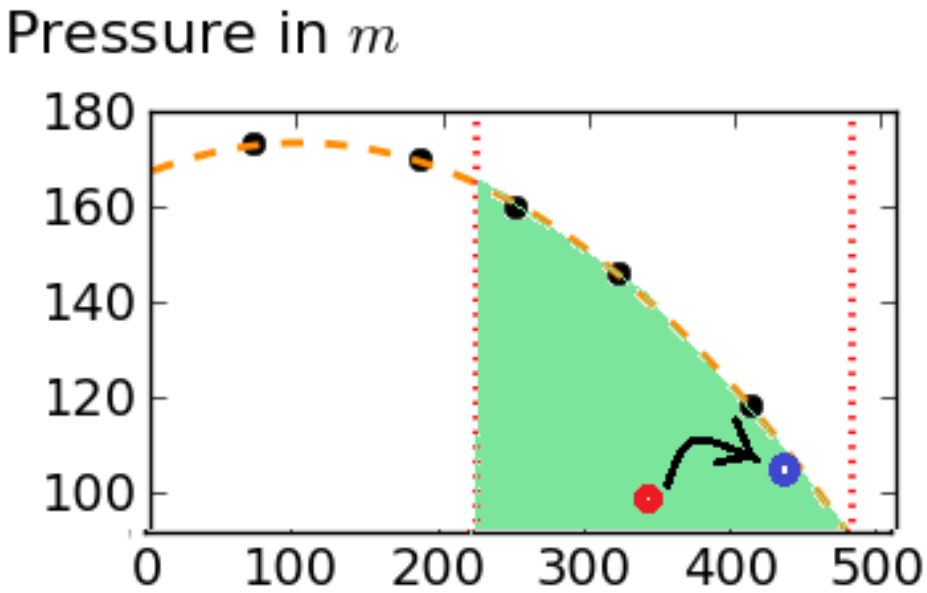
to a feasible solution

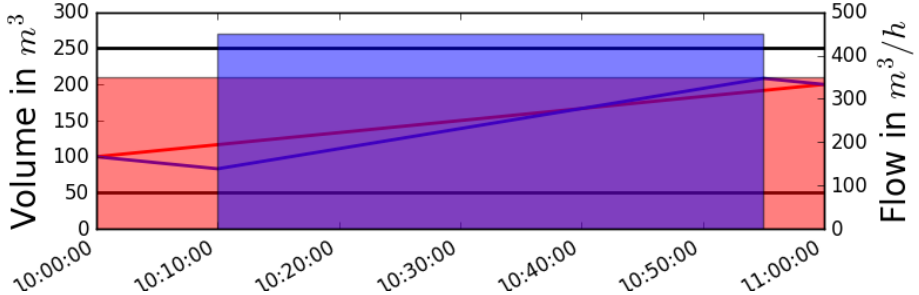
$$h_{jt} - h_{it} = \Delta H_{ij} - \gamma^1_{ij} q_{ijt}^{\gamma^2_{ij}} \text{ if } x_{ijt} = 1$$
$$Cost \leq \sum_{(i,j)\in P, t\in T} \Gamma(q_{ijt}, x_{ijt}) C_t \Delta_t$$

Why a convex relaxation ? (2/2)









From a relaxed solution...

$$h_{jt} - h_{it} \leq \Delta H_{ij} - \gamma_{ij}^{1} q_{ijt}^{\gamma_{ij}^{2}}$$
$$Cost = \sum_{(i,j)\in P, t\in T} \Gamma(q_{ijt}, x_{ijt}) C_t \Delta_t$$

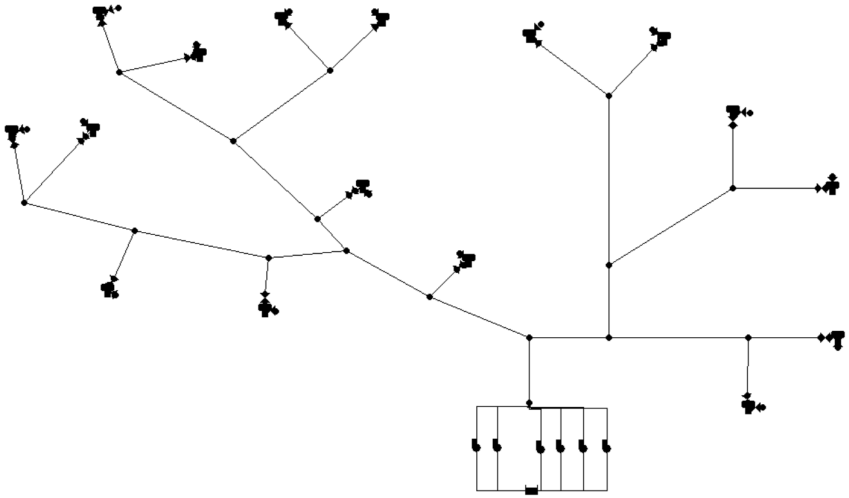
to a feasible solution

$$h_{jt} - h_{it} = \Delta H_{ij} - \gamma_{ij}^{1} q_{ijt}^{\gamma_{ij}^{2}} \text{ if } x_{ijt} = 1$$
$$Cost \leq \sum_{(i,j)\in P, t\in T} \Gamma(q_{ijt}, x_{ijt}) C_t \Delta_t$$

A branched rural network (1/2) : overview

Characteristics of the network

- Arborescent network
- Flow control valve before each reservoir
- Aggregated water demand of end-consumers at each reservoir



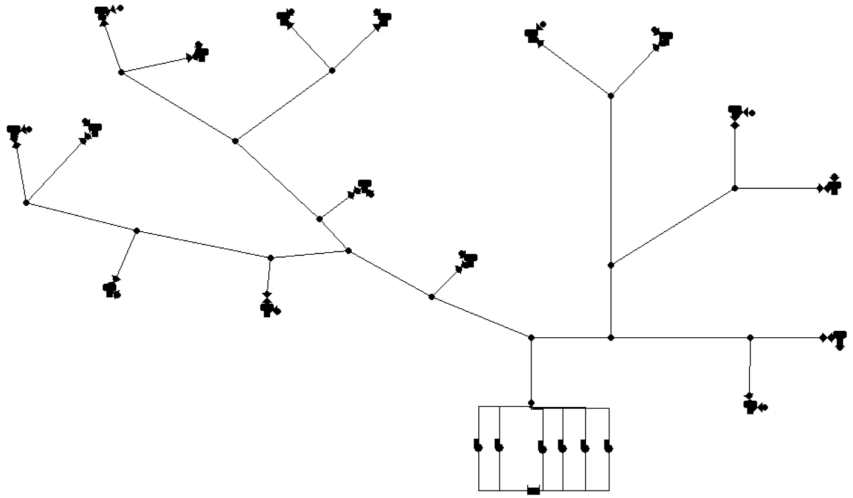
From relaxed to feasible solution

- *Proposition 1* - All pumps are identical : feasible solution at same cost
- *Proposition 2* - Otherwise : feasible solution with overcost bounded by a factor of $(\max_{k \in K} P_k - \min_{k \in K} P_k)$

A branched rural network (1/2) : overview

Characteristics of the network

- Arborescent network
- Flow control valve before each reservoir
- Aggregated water demand of end-consumers at each reservoir



From relaxed to feasible solution

- *Proposition 1* - All pumps are identical : feasible solution at same cost
- *Proposition 2* - Otherwise : feasible solution with overcost bounded by a factor of $(\max_{k \in K} P_k - \min_{k \in K} P_k)$

A branched rural network (2/2) : results

Evaluating the gains achieved

- 365 daily pump scheduling
- Comparison with current stratgey (BAU)
- 1 minute computational time
- Average optimality gap : 3%

Details in...

G. Bonvin, S. Demassey, C. Le Pape, N. Maïzi, V. Mazauric & A. Samperio. A convex mathematical program for pump scheduling in a class of branched water networks. Applied Energy, Elsevier, 2016, pp.In Press.

A branched rural network (2/2) : results

Evaluating the gains achieved

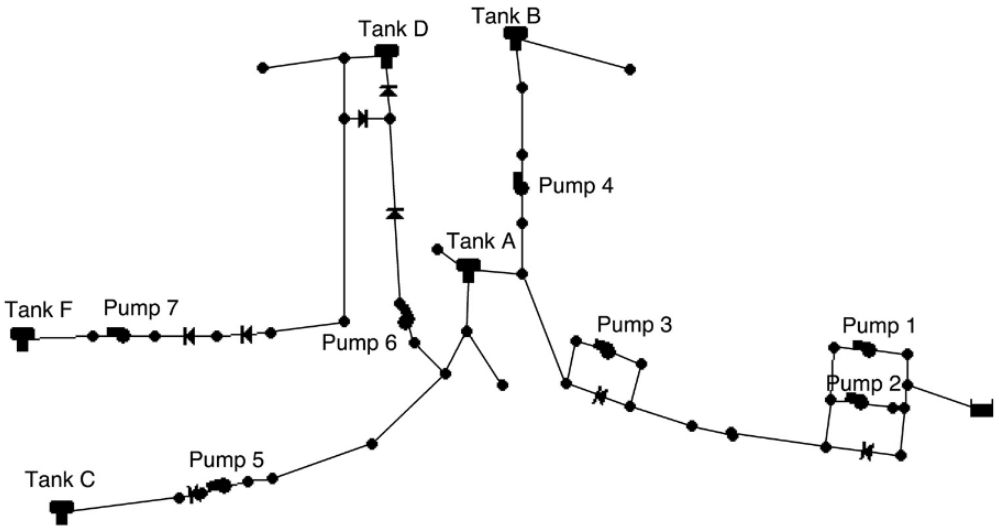
- 365 daily pump scheduling
- Comparison with current stratgey (BAU)
- 1 minute computational time
- Average optimality gap : 3%

	BAU	*MINLP*	ΔM/B
power cost $\sum_d z_d$ (k€)	58.7	48.8	-16.9%
power consumption $\sum_d \Psi_d Q_d \eta_d^{-1}$ (GWh)	1.335	1.157	-13.3%
mean discharge pressure $\frac{1}{365}\sum_d \psi_d$ (m)	133.1	103.9	-21.9%
mean pump efficiency $\frac{1}{365}\sum_d \eta_d$	74.2%	67.2%	-9.6%
mean power cost $\frac{1}{365}\sum_d C_d$ (€/MWh)	45.0	43.1	-4.2%

Details in...

G. Bonvin, S. Demassey, C. Le Pape, N. Maïzi, V. Mazauric & A. Samperio. A convex mathematical program for pump scheduling in a class of branched water networks. Applied Energy, Elsevier, 2016, pp.In Press.

Richmond network (1/2) : overview



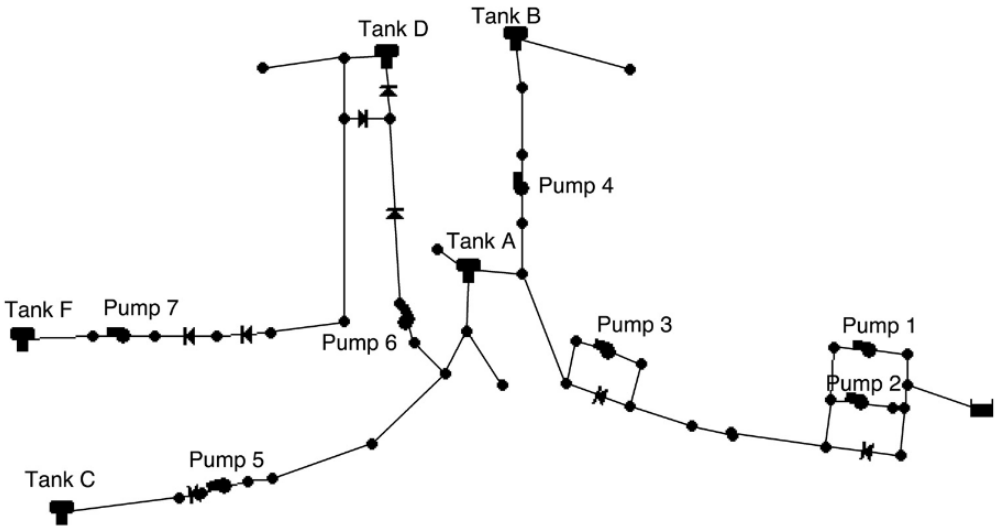
Characteristics of the network

- branched network
- No valves before tanks

From relaxed to feasible solution

- Adjustment of pump activity duration
- Pump activity delayed in case of volume constraints

Richmond network (1/2) : overview



Characteristics of the network

- branched network
- No valves before tanks

From relaxed to feasible solution

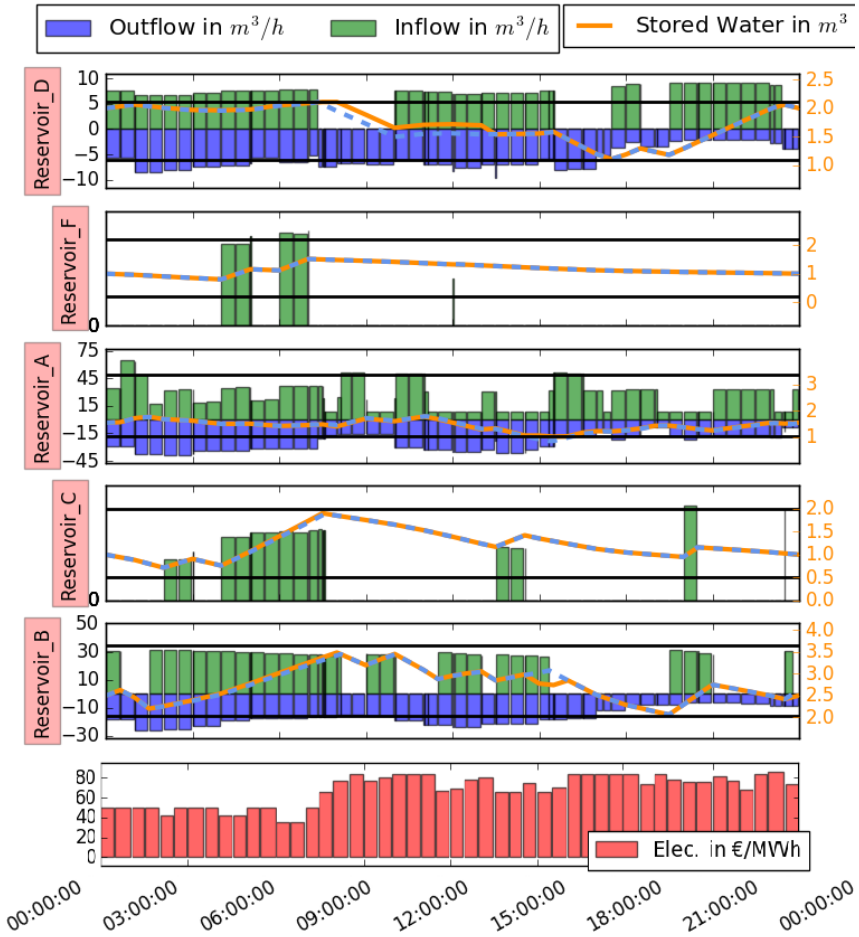
- Adjustment of pump activity duration
- Pump activity delayed in case of volume constraints

Richmond network (2/2) : results

Date	1st *sol*	*GAP* (900*s*)
21.05	43s	9.8 %
22.05	450s	9.3 %
23.05	476s	8.0 %
24.05	27s	8.5 %
25.05	29s	11.2 %
	205s	9.4%

Comparison with litterature

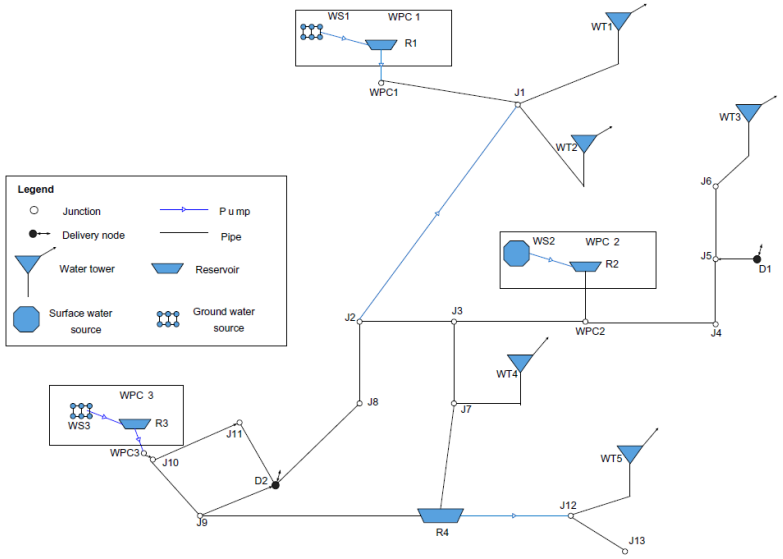
- Lagrangian decomposition (Ghaddar, 2015) : 12% en 5180 s
- Piecewise linearization (Ghaddar, 2015) : No solution in 2 hours
- Bender decomposition (Naoum-Sawaya, 2015) : quick solutions without optimality gap



De Watergroep network (1/2) : overview

Characteristics of the network

- Looped network
- Pipes with two-way flow
- Constraints on the head of intermediate nodes



From relaxed to feasible solution

- $\min \sum_{i \in P} M \left(z_{it} - z_{it}^0 \right) + \sum_{i \in R} \left(q_{it} - \left(q_{it}^0 - \frac{v_{i(t-1)} - v_{i(t-1)}^0}{\Delta t} \right)^2 \right)$

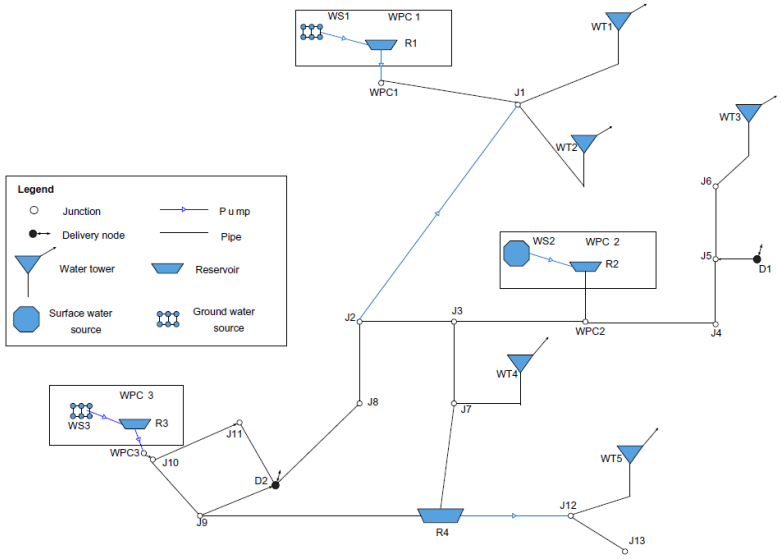
De Watergroep network (1/2) : overview

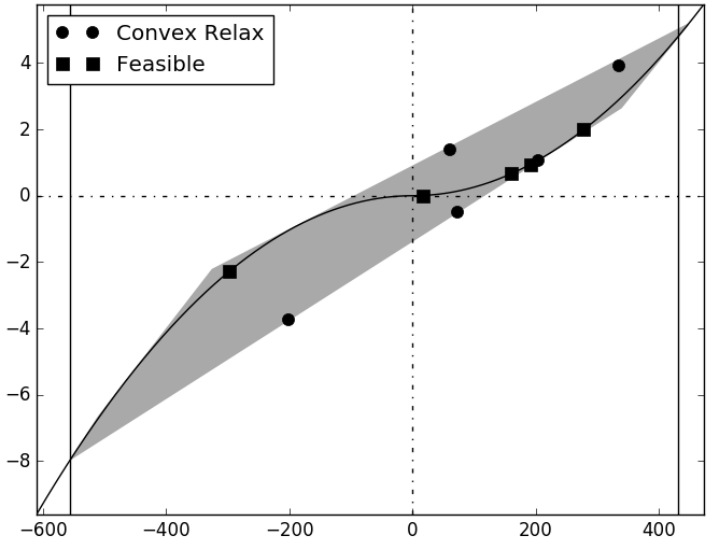
Characteristics of the network

- Looped network
- Pipes with two-way flow
- Constraints on the head of intermediate nodes

From relaxed to feasible solution

- $\min \sum_{i \in P} M \left(z_{it} - z_{it}^0\right) + \sum_{i \in R} \left(q_{it} - \left(q_{it}^0 - \frac{v_{i(t-1)} - v_{i(t-1)}^0}{\Delta t} \right)^2 \right)$





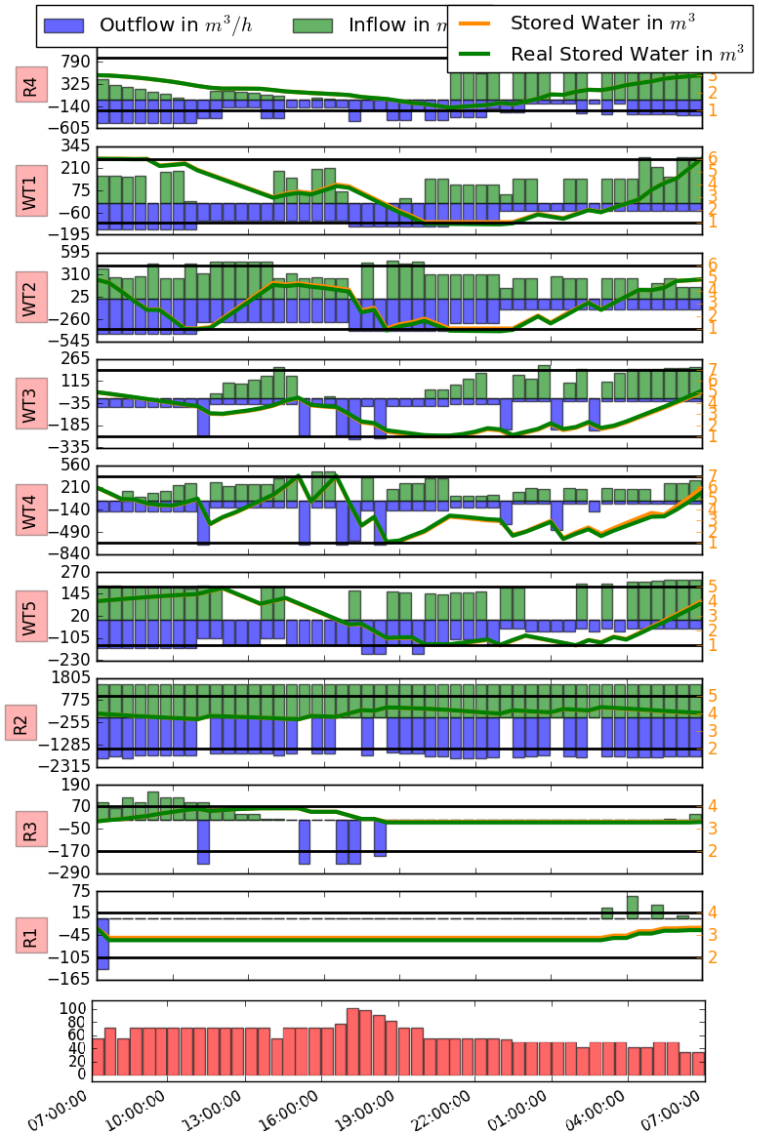
Réseau De Watergroep (2/2) : results

Linearization Verleye 2015 vs Relaxation

	Linearization		Relaxation	
T	$1^{st}sol$	$GAP(1h)$	$1^{st}sol$	$GAP(1h)$
5	7s	11s	<1s	<1s
12	169s	1.0%	4s	3025s
24	-	-	260s	2.0 %
48	-	-	1440s	3.4 %

Dynamic electricity tariffs

$Date$	$1^{st}sol$	GAP (1800s)
21.05	1240s	2.0 %
22.05	1010s	2.0 %
23.05	1294s	2.1 %
24.05	1010s	3.0 %
25.05	1707s	2.9 %
	1252s	2.4 %



Conclusion & Prospects

Pumping scheduling is almost convex !

- Pump power characteristics support minimization of pressure losses
- Relaxing pressure-related constraints produces near-feasible schedules

Results

- Significant annual electrical cost drops.
- Robustness to dynamic tariffs.

Perspectives

- Integrating water losses in the pumping scheduling process
- Designing pumping stations in term of global cost (investment + electrical costs)

Conclusion & Prospects

Pumping scheduling is almost convex !

- Pump power characteristics support minimization of pressure losses
- Relaxing pressure-related constraints produces near-feasible schedules

Results

- Significant annual electrical cost drops.
- Robustness to dynamic tariffs.

Perspectives

- Integrating water losses in the pumping scheduling process
- Designing pumping stations in term of global cost (investment + electrical costs)

Conclusion & Prospects

Pumping scheduling is almost convex !

- Pump power characteristics support minimization of pressure losses
- Relaxing pressure-related constraints produces near-feasible schedules

Results

- Significant annual electrical cost drops.
- Robustness to dynamic tariffs.

Perspectives

- Integrating water losses in the pumping scheduling process
- Designing pumping stations in term of global cost (investment + electrical costs)

Thank you for your attention !