Convex Relaxation for Water Distribution Systems

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Gratien Bonvin, Sophie Demassey

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1 Introduction
   • Pump scheduling in Water Distribution Systems (WDS)
   • Pumping stations and electricity markets

2 Convex relaxation in water distribution systems
   • Physical model
   • Why a convex relaxation?

3 Case studies
   • A branched rural network
   • Richmond network
   • De Watergroep network

4 Conclusion & Prospects
Water urban cycle, drinking water distribution systems and electricity consumption

Electricity consumption for Water Supply & Treatment in USA (EPRI 2002)

- Water supply:
  \( \sim 73 \text{ TWh (1.7\%)} \)
- Wastewater treatment:
  \( \sim 76 \text{ TWh (1.8\%)} \)

We focus on drinking water networks

- Electricity consumption:
  \( \sim 35 \text{ TWh (0.8\%)} \)
- Electricity consumed by pumping stations
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How to minimize electrical costs due to pumping stations?
Pumping stations and electricity markets

Current strategy: a step-by-step strategy driven by water volume thresholds

- Reservoir are filled to their maximal capacity during the night (off-peak tariffs)
- Reservoir are kept to their lower capacity during the day (peak tariff)

New challenges for the energy system and new opportunities for WDS

- Daily pump scheduling with day-ahed prices and electricity purchase on spot market (Ghaddar 2015)
- Increase the value of storage capacities with demand-response strategies (Menke 2016)
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Pumping scheduling: physical model

Flow equilibrium in nodes
\[ \sum_{(i,j)} q_{ijt} - \sum_{(j,k)} q_{jkt} = D_{jt} \]

Flow equilibrium in reservoirs
\[ V_{i(t+1)} - V_{it} = \left( \sum_{(j,i)} q_{ijt} - \sum_{(i,j)} q_{ijt} - D_{it} \right) \Delta_t \]

Head-flow relations for pipes and pumps
\[ h_{it} - h_{jt} = \kappa_{ij} \text{sign}(q_{ijt}) q_{ijt}^{\gamma_{ij}} \]
\[ h_{jt} - h_{it} = \Delta H_{ij} - \gamma_{ij}^{1/2} q_{ijt}^{\gamma_{ij}/2} \text{ si } x_{ijt} = 1 \]

Minimize electrical costs
\[ \text{Cost} = \sum_{(i,j) \in P, t \in T} \Gamma(q_{ijt}, x_{ijt}) C_t \Delta_t \]
\[ \Gamma(q_{ijt}, x_{ijt}) = A x_{ijt} + B q_{ijt} \]
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Why a convex relaxation? (1/2)

Mathematical issues

- Non-convex constraints for head-flow relations
- Binary variables for pump status

State-of-the-art methods

- Piecewise linear approximation
  (Morsi 2012, Geissler 2013, Verleye 2013/2015)
- Non-convex optimisation
  (Burgschweiger 2009, Ghaddar 2015)

Our proposition

- $h_{jt} - h_{it} = \Delta H_{ij} - \gamma_{ij}^1 q_{ijt}^2$ if $x_{ijt} = 1$
- $h_{it} - h_{jt} = \kappa_{ij} \text{sign}(q_{ijt}) q_{ijt}^{\gamma_{ij}}$
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- $h_{jt} - h_{it} \leq \Delta H_{ij} - \gamma_{ij}^1 q_{ijt}^2$
- $h_{it} - h_{jt} \in \text{Conv} \left( \kappa_{ij} \text{sign}(q_{ijt}) q_{ijt}^{\gamma_{ij}} \right)$
Why a convex relaxation? (2/2)

From a relaxed solution...

\[ h_{jt} - h_{it} \leq \Delta H_{ij} - \gamma_{ij}^1 q_{ijt} \gamma_{ij}^2 \]

\[ \text{Cost} = \sum_{(i,j) \in P, t \in T} \Gamma(q_{ijt}, x_{ijt}) C_t \Delta_t \]

to a feasible solution

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Cost \( \leq \sum_{(i,j) \in P, t \in T} \Gamma(q_{ijt}, x_{ijt}) C_t \Delta_t \)
A branched rural network (1/2) : overview

Characteristics of the network

- Arborescent network
- Flow control valve before each reservoir
- Aggregated water demand of end-consumers at each reservoir

From relaxed to feasible solution

- Proposition 1 - All pumps are identical : feasible solution at same cost
- Proposition 2 - Otherwise : feasible solution with overcost bounded by a factor of \((\max_{k \in K} P_k - \min_{k \in K} P_k)\)
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A branched rural network (2/2) : results

Evaluating the gains achieved

- 365 daily pump scheduling
- Comparison with current strategy (BAU)
- 1 minute computational time
- Average optimality gap : 3%

Details in...

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<table>
<thead>
<tr>
<th></th>
<th>BAU</th>
<th>MINLP</th>
<th>ΔM/B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power cost $\sum_d z_d$ (k€)</td>
<td>58.7</td>
<td>48.8</td>
<td>-16.9%</td>
</tr>
<tr>
<td>Power consumption $\sum_d \psi_d Q_d \eta_d^{-1}$ (GWh)</td>
<td>1.335</td>
<td>1.157</td>
<td>-13.3%</td>
</tr>
<tr>
<td>Mean discharge pressure $\frac{1}{365} \sum_d \psi_d$ (m)</td>
<td>133.1</td>
<td>103.9</td>
<td>-21.9%</td>
</tr>
<tr>
<td>Mean pump efficiency $\frac{1}{365} \sum_d \eta_d$</td>
<td>74.2%</td>
<td>67.2%</td>
<td>-9.6%</td>
</tr>
<tr>
<td>Mean power cost $\frac{1}{365} \sum_d C_d$ (€/MWh)</td>
<td>45.0</td>
<td>43.1</td>
<td>-4.2%</td>
</tr>
</tbody>
</table>

Details in...

Richmond network (1/2) : overview

Characteristics of the network
- branched network
- No valves before tanks

From relaxed to feasible solution
- Adjustment of pump activity duration
- Pump activity delayed in case of volume constraints
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<table>
<thead>
<tr>
<th>Date</th>
<th>1st sol</th>
<th>GAP (900s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.05</td>
<td>43s</td>
<td>9.8 %</td>
</tr>
<tr>
<td>22.05</td>
<td>450s</td>
<td>9.3 %</td>
</tr>
<tr>
<td>23.05</td>
<td>476s</td>
<td>8.0 %</td>
</tr>
<tr>
<td>24.05</td>
<td>27s</td>
<td>8.5 %</td>
</tr>
<tr>
<td>25.05</td>
<td>29s</td>
<td>11.2 %</td>
</tr>
<tr>
<td></td>
<td>205s</td>
<td>9.4 %</td>
</tr>
</tbody>
</table>

Comparison with literature

- Lagrangian decomposition (Ghaddar, 2015) : 12% en 5180 s
- Piecewise linearization (Ghaddar, 2015) : No solution in 2 hours
- Bender decomposition (Naoum-Sawaya, 2015) : quick solutions without optimality gap
De Watergroep network (1/2) : overview

Characteristics of the network

- Looped network
- Pipes with two-way flow
- Constraints on the head of intermediate nodes

From relaxed to feasible solution

\[
\begin{align*}
\min \sum_{i \in P} M (z_{it} - z_{it}^0) + \\
\sum_{i \in R} \left( q_{it} - \left( q_{it}^0 - \frac{v_i(t-1) - v_i^0}{\Delta t} \right) \right)^2
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Réseau De Watergroep (2/2) : results

Linearization Verleye 2015 vs Relaxation

<table>
<thead>
<tr>
<th>$T$</th>
<th>$1^{st}$ sol</th>
<th>GAP (1h)</th>
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<tbody>
<tr>
<td>5</td>
<td>7s</td>
<td>11s</td>
<td>&lt;1s</td>
<td>&lt;1s</td>
</tr>
<tr>
<td>12</td>
<td>169s</td>
<td>1.0%</td>
<td>4s</td>
<td>3025s</td>
</tr>
<tr>
<td>24</td>
<td>-</td>
<td>260s</td>
<td></td>
<td>2.0%</td>
</tr>
<tr>
<td>48</td>
<td>-</td>
<td>1440s</td>
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<td>3.4%</td>
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Dynamic electricity tariffs

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<tbody>
<tr>
<td>21.05</td>
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<td>2.0%</td>
</tr>
<tr>
<td>22.05</td>
<td>1010s</td>
<td>2.0%</td>
</tr>
<tr>
<td>23.05</td>
<td>1294s</td>
<td>2.1%</td>
</tr>
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<td>1010s</td>
<td>3.0%</td>
</tr>
<tr>
<td>25.05</td>
<td>1707s</td>
<td>2.9%</td>
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Conclusion & Prospects

Pumping scheduling is almost convex!

- Pump power characteristics support minimization of pressure losses
- Relaxing pressure-related constraints produces near-feasible schedules

Results

- Significant annual electrical cost drops.
- Robustness to dynamic tariffs.

Perspectives

- Integrating water losses in the pumping scheduling process
- Designing pumping stations in term of global cost (investment + electrical costs)
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