# What drives Professional services firms prices? *

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June 28, 2018

**Abstract**

In this paper, I propose a competitive equilibrium in which a matching occurs between high level quality PSFs and large clients. I demonstrate that this means that PSFs price variations only depends in 4 variables: the number of potential local clients, their average revenue, the number of competing PSFS and the number of PSFs employees. I finally show that across 26 OECD countries and over the last 10 years this model can explain more than 90% of PSFs price variations across country and time.

2010 *JEL Classification.* L84;L14;D40;L15;C78
*Keywords and phrases.* Professional Services; Services Quality; Pricing; Matching theory

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*Important Legal Remarks. The findings and opinions expressed in this paper are those of the authors and do not reflect any positions from any company or institution.

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# 1 Introduction

Amongst the OECD, professional services firms [PSFs] prices (defined as the ratio of PSFs production over the number of employees adjusted by the purchasing power parity [PPP]) have been exhibiting significant variations across both countries and time over the past decade. This can be perceived as a risk that PSFs have to mitigate in their strategic planning activities.

There is however a scarce empirical literature on PSFs prices, which, according to a recent meta analytic study (Fuentes & Porcuna, 2016) features three main type of explanatory variables for prices fluctuations: the client complexity (associated to its size), the services quality and the length of the client relationship (Lassala, Carmona, & Momparler, 2016). On the other hand, there is an abundant theoretical economic literature on the matching mechanisms that associate high quality or high reputation agents (e.g. workers) with a given type of principal (e.g. firms) (Anderson & Smith, 2010).

I therefore build upon the matching model of (Gabaix & Landier, 2008) to propose a micro level explanation of PSFs prices that is consistent with a positive assortative matching between client size and service quality. I then show that this model can be integrated at a macro economic level to explain more than 90% of the cross country and time PSFs prices differences within the OECD.

I contribute to the PSFs literature by providing a competitive prices equilibrium model and a consistent micro and macro level view of PSFs prices mechanisms. I estimate the model parameters on the main PSFs domains: management consulting, accounting and auditing, legal and engineering to put the traditional literature auditing focus in perspective.

# 2 Model.

## 2.1 General Framework.

Assume that PSFs workers can offer services of quality $q \in [Q_{min}; Q_{max}]$. Let be $h(q, z)$ the number of PSFs employing $z$ individuals and that have a service quality $q$. Conversely, assume that clients have a size $s \in [S_{min}; S_{max}]$ and call $f(s)$ the number of clients of size $s$. Clients of size $s$ generate a profit in period 0 $\pi_0(s)$, with $\pi_0'(s) > 0$. To optimize their profits in period 1 $\pi_1(s)$, they can contract PSFs workers of quality $q$ for a price $P(q)$, so that:

$$\pi_1(s) - \pi_0(s) = \pi_0(s)^\gamma.\Theta.q - P(q) \tag{1}$$

Where $\gamma > 0$ (resp. $\Theta$) is a scaling factor that accounts for differences in services return with respect to clients profits (resp. for clients profit increase with service quality). The problem for a client of size $s$ therefore is:

$$\max_q(\Theta.\pi_0(s)^\gamma.q - P(q)) \tag{2}$$

We now turn to the characterization of the competitive equilibrium associated to this problem. This consists of an assignment function $M : [S_{min}; S_{max}] -> [Q_{min}; Q_{max}]$ which pairs clients of size $s$ with workers of quality $M(s)$ and of price function $P(q)$. The equilibrium is efficient if it maximizes the overall surplus generated by services (e.g.$\int_0^S \pi_0(s)^\gamma.M(s).f(s)ds$) subject to resource constraints.

**Proposition 2.1** *If $\partial_s(\pi_0(s)^\gamma.f(s)) \geq 0$, there is a positive assortative matching at equilibrium (i.e larger clients get better services) and:*

$$M(s) = E^{-1}(F(s))$$

*Where* $F(s) = \int_s^{S_{max}} f(x)dx$ *and* $E(q) = \int_q^{Q_{max}} \int z.h(z,x)dzdx$*. Otherwise, if* $\partial_s(\pi_0(s)^\gamma.f(s)) < 0$*, there is a negative assortative matching and:*

$$M(s) = E^{-1}(F(S_{min}) - F(s))$$

The proposition (2.1) naturally generates the following maximization conditions from (2):

$$\Theta.(\pi_0(M^{-1}(q))^\gamma.q)' = P'(q) \leftrightarrow P(q) = \int_{Q_{min}}^{q} \Theta.(\pi_0(M^{-1}(x))^\gamma.x)'dx + P(Q_{min}) \quad (3)$$

### 2.2 Resolution with power laws.

Assume that clients are distributed according to a power law $F(s) = \hat{F}.s^{\alpha+1} = \int_s^\infty f(x)dx$ ((Axtell, 2001)), that profit follows a Cobb-Douglas type of function $\pi_0(s) = \Pi.s^\beta$ ((Ribes, 2018)) and that quality is distributed according to $h(z,q) = h_0.z^{\eta(\hat{E})}.q^\nu$. Note that this means that the average quality of service in independent of firm size as $\frac{\int q.h(z,q)dq}{\int h(z,q)dq} = \frac{\nu+1}{\nu+2}$. This also links the total employment $\hat{E} = \int\int z.h(z,q)dqdz$ and that the total number of PSFs $\hat{G} = \int\int h(z,q)dqdz$, so that whether or not is $\eta(\hat{E}) < -2$:

$$\hat{G} = \hat{E}.A.\frac{(\eta(\hat{E})+2)}{(\eta(\hat{E})+1)}$$

Where for $z_{max}$ large enough, $A = 1$ in the case $\eta(\hat{E}) < -2$ or $A = 1/z_{max}$ otherwise. Assuming that $\hat{G} = C.\hat{E}^\rho$ holds (see 3.2), this means that:

$$\eta(\hat{E}) = \frac{(2 - C.\hat{E}^{\rho-1})}{(C.\hat{E}^{\rho-1} - 1)} \quad (4)$$

The client - PSFs matching at equilibrium is thus given by:

$$(M(s))^{\nu+1}.\frac{-h_0}{(\nu+1)(\eta(\hat{E})+2)} = \hat{F}s^{\alpha+1} \leftrightarrow M(s) = (\frac{\hat{F}.(\eta(\hat{E})+2)}{\hat{G}.(\eta(\hat{E})+1)})^{\frac{1}{\nu+1}}.s^{\frac{\alpha+1}{\nu+1}}$$

Therefore the overall PSFs sector revenue $R$ is:

$$R = \int P(s)f(s)ds = \int \Theta.\Pi^\gamma.s^{\gamma.\beta}(\frac{\hat{F}}{\hat{E}})^{\frac{1}{\nu+1}}.s^{\frac{\alpha+1}{\nu+1}}\hat{F}s^\alpha.ds$$

And as $\frac{\eta(\hat{E})+2}{\eta(\hat{E})+1} = C\hat{E}^{\rho-1}$, it comes that the average rate per employee $RpE = \frac{R}{E}$ obeys:

$$(\frac{RpE}{\hat{F}}) = \hat{C}.\Pi^\gamma.(\frac{\hat{F}}{\hat{G}})^{\frac{1}{\nu+1}}.(\hat{E})^{\frac{\rho-1}{\nu+1}-1} \quad (5)$$

# 3 OECD PSFs prices

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### 3.1 OECD Data.

The model analysis relies on the OECD Structural Statistics on Industry and Services (SSIS) database according to the 4th revision of the International Standard Industrial Classification (ISIC Rev. 4). Out of this database, four main time series where extracted. The variable "production" was used as a proxy for PSFs revenue $R$,the "number of enterprises" as $\hat{F}$ and the number of competitors $\hat{G}$, the "total employment" as $\hat{E}$ and to proxy $\Pi$, the actual sector production divided by the sector number of firms was used. Under a power law setting, a sector production is indeed $\int \Pi.s^{\beta+\alpha}.f_1 ds \propto \Pi.\hat{F}$.
This resulted in time series for 26 countries, namely: Austria, Czech Republic, Estonia, Finland, Hungary, Latvia, Luxembourg , New Zealand, Norway, Poland, Portugal, Slovenia, Belgium, Denmark, Germany, Ireland, Italy, Netherlands, Slovak Republic, Spain, Sweden, United Kingdom, Greece, Turkey, France, Iceland. The extracted time series represented a timespan of 10 years (from 2007 to 2016). Finally to allow a cross country comparison, all the production variables from the OECD were adjust by the country purchasing power parities [PPP] for GDP, which normalizes the local production in local currency into an equivalent amount in US$ while accounting for differences in purchasing power.
After clean up, the dataset consisted in $n = 186$ points (one point per country per year). Each point $n$ consisted of 5 coordinates $(\hat{F}_n, \hat{G}_n, R_n, \Pi_n, \hat{E}_n)$. Once PPP adjusted, the average revenue generated per employee in the PSFs sector was of 216k$ per year with a standard deviation of 85.5k$ per year. There was an average of 866k clients ($\hat{F}$) (std. 965k) for an average of 152k PSFs ($\hat{G}$) (std. 171k). The average client production $\frac{\Pi.(\alpha+1)}{\beta+\alpha+1}$ was of 1.95 M$ per year (std. 1.18M$).

### 3.2 Model estimation.

Of the entire OECD PSFs landscape, management consulting ["MC"] represents on average 20% of the total PSFs within each country, the legal and accounting domain about 29% and the architectural and engineering ["AE"] domain an average 23%. First, for each domain, I validated the equation (4) by running:

$$log(\hat{G}_n) = \rho. \log(\hat{E}_n) + log(C) + \epsilon_n \tag{6}$$

| Parameter | MC estimates | Legal estimates | Accounting estimates | AE estimates |
|---|---|---|---|---|
| $\rho$ | 0.89 (0.02) | 0.69 (0.05) | 0.83 (0.04) | 0.94 (0.051) |
| $ln(\hat{C})$ | 0.4 (0.29) | 2.45 (0.48) | 0.88 (0.47) | -0.20 (0.55) |
| $R^2$ [%] | 85.5 | 53.3 | 66.5 | 66.4 |

Table 1: Model (6) Calibration results on OECD countries.

Then I ran the following regression analysis:

$$\log(\frac{RpE_n}{F_n}) = \frac{1}{\nu+1}.ln(\frac{\hat{F}_n}{G_n}) + \gamma.ln(\Pi_n) + ln(\hat{C}) + (\frac{\rho-1}{\nu+1} - 1).log(E_n) + \epsilon_n \tag{7}$$

Results are summarized in the table (2).

From a comparison standpoint, defining the average daily rate $DR$ of one type of service as the average

| Parameter | MC estimates | Legal estimates | Accounting estimates | AE estimates |
|---|---|---|---|---|
| $\frac{1}{\nu+1}$ | -0.56 (0.03) | -0.37 (0.06) | -0.28 (0.09) | -0.29 (0.12) |
| $\gamma$ | 0.87 (0.05) | 1.26 (0.07) | 1.19 (0.05) | 0.79 (0.06) |
| $ln(\hat{C})$ | -4.21 (0.21) | -5.41 (0.39) | -4.87 (0.32) | -4.64 (0.57) |
| $\frac{\rho-1}{\nu+1}-1$ | -0.87 (0.05) | -0.89 (0.02) | -0.96 (0.03) | -0.91 (0.03) |
| $R^2$ [%] | 94.6 | 88.7 | 92.2 | 86.1 |

Table 2: Regression (7) calibration results on OECD countries across PSFs domains

annual RpE reported on 1750 worked hours per year times 8 hours worked per day, it comes that PPP adjusted:

$$DR_L = 1039.7\$/day > DR_{MC} = 1008.3\$/day > DR_{AE} = 800.3\$/day > DR_A = 534.3\$/day$$

From a client standpoint, both MC and AE services yield more return for small firms as $\gamma < 1$, which is not the case for legal and accounting services. From a price sensitivity to the overall domain employment ($\frac{\rho-1}{\nu+1} - 1$),all the domains appears to be similar with a $\frac{\rho-1}{\nu+1} \approx 0.1$. Finally from a quality distribution standpoint, the scarcity of quality appears very marked in accounting and in AE but less so in the legal and management consulting domains.

## 4 Conclusion

In this paper, I have shown that within OECD countries, more than 90% of PSFs average rate per employee variations can be explained with 3 variables: the number of competitors within the country, the potential number of clients and the available number of PSFs employees. I have demonstrated that the observed relationship on PSFs prices variations within countries can be explained by a quality based matching mechanism between PSFs and clients: the largest clients are getting the best services as this maximizes the overall economic surplus within a country. When thinking in terms of quality based matching, the OECD data shows that high quality is rare among PSFs, that at equal service quality, the return of PSFs services are lower for larger firms.
In terms of future area of research, it would be interesting to see how the observed relationship can be evolved to take into account differences among clients industrial sectors. This could notably help understanding how PSFs competition is organized among clients, if PSFs services return are always more important for small clients and to which extend prices change across sector for the same type of service.

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