A model of professional service firms performance with an application to US accounting firms

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Abstract

This paper models the dynamics of professional services firms performance. To do so, it first describes how those firms produce their specific outputs and how their productivity drives competitiveness. The associated equilibrium in then used to propose profit optimal organic and inorganic growth strategies, which are then discussed to account for the effect of potential future technological disruption.
Applying the proposed theory to US accounting firms explains the differences in performance observed between large (i.e. more than 50 partners) and small (i.e. less than 50 partners) accounting partnerships and suggests that small (resp. large) US accounting partnerships mainly grow inorganically (resp. organically).

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1 Introduction

Professional services are commonly defined (see [Empson et al., 2015]) as the sum of legal, accounting, management consulting, advertising, engineering and architectural, scientific research and veterinary services[1]. As per the US census, professional services have grown by 4.2% in revenue year on year (on average) between 2008 and 2018, outperforming the manufacturing sector [NAICS code 31 to 33], which year on year revenue growth has been on average of 2.1% over the same period. On the other hand, employment in professional services across the US has grown by 0.49% on average per quarter, while the manufacturing sector has shown an employment reduction of about -0.22% per quarter.
At an aggregated level, those preliminary statistics stress that the manufacturing sector, on which the economic and management literature has classically been focused, and professional services obey different dynamics. Those differences have been acknowledged by the academic community in the past decade and have led to the development of a small research niche, nowadays anchored in the field of industrial organization by the seminal works of [Løwendahl, 2005] and [Maister, 2012]. If the performance and dynamics of manufacturing firms is a thoroughly discussed topic (see [Bottazzi and Secchi, 2003] for instance), there has not been to my knowledge any adaptation made specifically for professional services firms [referred to as PSFs in the rest of this paper]. This appears as a gap, which can be easily

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[1]Please refer to the US census website for high level statistics on the professional services sector, accessible within the North American Industrial Classification System (i.e. NAICS) through code 54

brought to life through a few examples. For instance, manufacturing firms mostly rely on technology (and therefore capital) to produce a good, while PSFs mostly rely on labor to tailor a service. This naturally raises the question of the impact of the difference in the production engine on firms long term profitability. Similarly, if manufacturing firms rely on capital while PSFs rely on human capital, this has to entail a different growth strategy. Coming up with such examples is not difficult and this paper will start to start to bridge the associated gap.

To explain the performance patterns of PSFs, this paper will build upon three main strands of the economic literature. First it builds upon the theory of firm growth. Since the seminal empirical work of Gibrat (see [Gibrat, 1931]), which was later completed by ([Evans, 1987]), it has been thoroughly demonstrated that the growth rate of firms decreases as they become larger and that their performance becomes more predictable as they age. Additionally the larger and older firms are, the more likely they are to survive. From this empirical work has then stemmed a host of theories aimed at explaining firms' development ([Stanley et al., 1996],[Coad et al., 2017],[Haltiwanger et al., 2013]...) and survival ([Anyadike-Danes and Hart, 2014],[van Stel et al., 2017],[Cowling et al., 2018]...). The center piece of this research stream is nowadays Hopenhayn's framework ([Hopenhayn, 1992]), which explains both growth and survival as the result of productivity shocks. However most of the growth studies focus on manufacturing and little has been done with respect to the service industry ([Nassar et al., 2014]), even though it has been acknowledged that performance mechanism are industry dependent ([Mansfield, 1962]) [2]. This is therefore the first area of contribution for this paper.

Second, this work contributes to the academic literature related to professional services firms. To date, PSFs have not attracted a lot of research because of two of their structural characteristics. First, as PSFs are mainly organized as privately held partnerships ([Maister, 1982]), their performance is rarely publicly reported. Second, as PSFs are diversified ([Greenwood et al., 2005]) and usually supply client-specific combinations of services, their output is not as easy to measure for an outside observer as a unit of good. To overcome those difficulties and capture the performance patterns specific to this industry, the academic community has so far used PSFs' reputation as the input to predict performance (see [Bar-Isaac et al., 2008], [Bar-Isaac et al., 2012]). Reputation is however difficult to assess consistently and the community still considers the professional services sector to lack a holistic theory (see [Skjølsvik et al., 2017]). This paper addresses this gap by exploring competitiveness at a service level and by constructing an "insider" picture of PSFs performance as portfolios of services [3].

Finally, this work builds upon the literature on continuous optimization. It leverages Hamilton-Jacobi-Bellman [HJB] frameworks (see [Intriligator, 2002]), which are known to offer a suitable formalization of profit maximization/cost minimization problems in economy (see [Doumic et al., 2017] for an example). This paper provides an analytical solution of a PSF specific HJB model, which sits at the frontier between operational research and economy. This appears unique and innovative as there has not been previously any formalized PSFs' performance theory, let alone one which uses continuous tools.

The remaining of this paper is organized as follows. Section (2) provides a descriptive analysis of the delivery of professional services. This is then leveraged in section (3) to model PSFs revenue in a competitive landscape. This model is afterwards used first to design both organic (section (4)) and inorganic (section (5)) profit-maximizing growth strategy for PSFs and second to understand the

[2] Within the professional services sector, some empirical evidences have even shown that some sub sector may not even follow Gibrat's law (see [Galanter and Palay, 1990] for an example on law firms)

[3] Expert services are indeed provided by partners who employs around 10 individuals (see [Zerni, 2012] for an accounting example). A firm that employs more than 500 individuals in a given location may therefore be considered as a portfolio of more than 50 specialties.

impact of technological investment on PSFs performance (section (6)). Finally section (7) discusses potential elements that can explain performance heterogeneity across firms. Note that throughout the paper, the proposed theory is illustrated by examples pertaining to US accounting firms and that additional areas of research are highlighted in the conclusion (section (8)).

2 Delivering professional services.

Labor in PSFs is made of production and sales activities (see [Løwendahl, 2005]). Selling is a complex task as it means tailoring a credible promise to a network of prospective clients. On the other hand, production can be scripted to follow a plan and therefore can be supported by resources with less expertise. As a result, labor in professional services is allocated across two types of workers, often coined "finders" and "grinders" (see [Maister, 2012]).

It takes a considerable amount of time for an individual to develop a recognized expertise in the professional services field and become a "finder" (i.e. 10 to 15 years (see [Morris and Pinnington, 1998])). This has led professional services firms to be mostly structured as privately owned partnerships. This form indeed allows "finders" to fully ripe the benefits of their expertise as equity partners, while providing "grinders" with a natural incentive structure to develop their expertise (see [Greenwood et al., 1990]). The concept of a professional services firm therefore emerges from a labor distribution across S partners that can both sell and produce services and E employees that can only produce services under the supervision of an expert.

Within a PSF, a partner's revenue R is generated by the amount of client $N(L)$ to which he can sell services to at a price p when supported by L employees. To increase his revenue, a partner must therefore delegate a fraction $f(L)$ ($\in [0;1]$) of the production activities of his $N(L)$ clients to focus on selling activities. Both partners and employees have an availability of τ units of time and it can be assumed that sales (resp. delivery) activities take T_{sales} (resp. T_{prod}) units of time per client. Additionally, service delivery needs to be coordinated, which means that employees have to spend a fixed portion of time T_{coord} with each other to articulate an output (see [Dunbar, 1993] for a discussion). Among a pool of L workers, the first one has therefore to connect to $L-1$ individuals, the second worker to $L-2$ individuals, so that overall the amount of time spent on coordination activities is given by: $T_{coord}.\sum_{1\leq l\leq L}(L-l)$. This means that coordination/communication between individuals is associated to quadratic costs, which is a classic assumption in economic models (see for example [Crawford and Sobel, 1982] or [Dessein, 2002]) and that coordination limits the delegation of production activities (see [Williamson, 1967] for a discussion).

As a result, the amount of clients a partner can manage obeys the following rules:

$$\tau = \overbrace{N(L).T_{sales}}^{\text{Sales activities}} + \overbrace{T_{prod}.N(L).(1-f(L))}^{\text{Production activities}} \tag{1}$$

$$L.\tau = \overbrace{N(L).f(L).T_{prod}}^{\text{Delegated production activities}} + \overbrace{T_{coord}.\sum_{1\leq l\leq L}(L-l)}^{\text{Coordination activities}} \tag{2}$$

Calling $\alpha = \frac{p.\tau}{T_{sales}+T_{prod}}$ the labor productivity of a partner and $\gamma = \frac{T_{coord}}{2.\tau}$ the fraction of working time employees spend on coordination, the system ((1)-(2)) yields that the revenue R a partner generates is given by:

$$R(L) = \alpha.(1 + L(1+\gamma) - \gamma.L^2) \tag{3}$$

It has to be noted that this production system is yet subject to two constraints. First, a partner must have enough resources to service at least one client. Second a partner can not delegate more work than already available. This means that the number of employees a partner has is bounded between $\underline{L}$ and $\overline{L}$ (see lemmas (9.1) and (9.2) in the appendix for functional forms). Finally this model of production has two notable effects on firm's standard productivity indicators:

Lemma 2.1 *Assuming that the partnership size remains unchanged, when partners recruit new employees, their firm's total labor productivity (ρ), defined as the revenue generated by the firm S partners divided by the amount of work realized across both the partners and employees population (i.e. $\rho(L) = \alpha.(1 - \gamma.L.\frac{(1-L)}{1+L})$), decreases, while the revenue per employee ($\frac{R}{E} = (\frac{1}{L} + (1-\gamma) + L.\gamma).\alpha$) increases if delegation is sufficient (i.e. $L \geq \sqrt{\frac{1}{\gamma}}$).*

Example. The assumptions of this production model can be tested against real world data. The numbers of partners and employees as well as PSFs' revenue are for instance published by the *Public Accounting Report* on a yearly basis for the top 100 US accounting firms. This report covers about 75% of total US employment in the sector. Its 2018 edition showcases an average number of 9.1 (3.2) employees per partner and a revenue of 1.9 (0.9) M$ per partner. It shows that across all accounting firm, increasing delegation leads to a linear revenue improvement (see figure (1)) and that albeit crude, this type of model explains $R^2 = 96\%$ of the observed revenue variance in the accounting report. The estimation of the parameters of equation (3) via OLS shows that for every new employee, partner's revenue increases by $\alpha = 0.18$M$ per year (0.007) and that across the overall panel of US accounting firms coordination does not appear to have a significant impact on partner's revenue ($\alpha.\gamma = -0.0003$ (0.0004)).

Section (2) has therefore presented a model of service production, which appear credible against real world data. However the model does not explain how prices are set nor why there are important delegation differences within PSFs (within the 2018 *Public Accounting Report* the number of employees per partner L indeed ranges from 5 to 35). The proposed model will therefore be completed in section (3) to capture how professional services prices (and ultimately partners' revenue) result from a competitive equilibrium before being used alongside partners' profit maximization objectives to explain how delegation evolves as PSFs grow in sections (4) to (6).

3 Competition in professional services.

As seen in section (2), partners' revenue (i.e. R) depends in both the price of the service they offer (i.e. p) and the number of client they can manage (i.e. N). If the previous section has shown that partners' bandwidth (i.e. N) is a function of the number of employees (i.e. L) they delegate production activities to, prices fluctuations need to be explained. The classic mechanism used in economic theory to explain how prices are set is to consider an equilibrium where firms compete with each other based on their productivity (see ([Hopenhayn, 1992])). In the case of PSFs, this means that the price p at which a PSF goes to market results from its delivery speed against the ones of its competitors.

As PSFs are known to continuously tailor their service offerings and adjust their delivery speed as their partnerships evolve [4] (i.e. service production speed $T_{prod}(S)$ is a function of the partnership

[4]This occurs either through specialization (for instance on an industrial sector or a specific offer) or through cross selling (selling an integrated package with multiple services to a client) (see [Greenwood et al., 2005] for a review)

size) , they therefore come to clients (through requests for proposals for instance) with a price $p(S)$ representative of their partnership structure. This price gets communicated to clients, who get a value V for the service and choose the provider who offers the most productive alternative (i.e. maximizes the client's gains for a given delivery time), a choice that can be articulated as the following maximization problem:

$$\max_S \frac{V.\tau}{T_{sales} + T_{prod}(S)} - \alpha(S) \tag{4}$$

This decision criteria can be used to infer the price structure of the professional services sector (see appendix for a proof).

Proposition 3.1 *If production speed increases (i.e. $\partial_S T_{prod} \geq 0$) [resp. decreases] with the size of the partnership, the price of the associated services increases ($\partial_S p \geq 0$) [resp. decreases] and so does partners' productivity ($\alpha(S)$), as professional services prices are given by:*

$$\forall S \geq S_{min} \qquad p(S) = V - \frac{T_{sales} + T_{prod}(S)}{T_{sales} + T_{prod}(S_{min})}.(V - p(S_{min})) \tag{5}$$

Proposition (3.1) appears consistent with empirical evidences reported in the literature. It was indeed noted that tailored offerings (which takes longer to produce) in accounting ([Zerni, 2012], [Mayhew and Wilkins, 2003], [Palmrose, 1986], [Craswell et al., 1995]) and law firms ([Hadfield, 1999]) led to higher prices. In essence, this means that the less commoditized the service, the higher its price, which is in line with the seminal ideas of [Maister, 1982].

Special case. Recent empirical evidences can be used to propose an analytical form for professional services prices and partners' revenue. On one hand, labor productivity has been shown to be distributed across firms according to a power law (see [Aoyama et al., 2009]). This means that the number of partnerships that have a productivity α is $\propto \alpha^{\frac{1}{\phi}}$. On the other hand, it is also known that firms distribution follow a zipf law with respect to their size (see [Axtell, 2001]) The number of partnerships with S partners can then be assumed to be proportional S^{-1}. Those two findings therefore implies that partners' productivity follows a power law in their firm's partnership size (i.e. $\alpha(S) \propto S^{-\phi}$). As service prices and partners' productivity are linked (see appendix), professional services prices and partners' revenue can be analytically expressed as:

$$p(S) = \frac{p(S_{max}).V}{p(S_{max}) + (V - p(S_{max})).e^{-\phi.(S_{max}-S)}} \qquad R(S, L) = \alpha_0.S^{-\phi}.(1 + (1+\gamma).L - \gamma.L^2) \tag{6}$$

Example. The functional form (6) can be tested against the 2018 US *Public Accounting report* data set, which was used in section (2). To do so, a simple two step approach was used. First, for a given productivity elasticity (i.e. ϕ), the sensitivity of a partner's revenue to its leverage model (α_0,γ) was estimated. The procedure was then iterated over a range of value for ϕ (i.e. $\phi \in]-2; 2[$ with a step of 0.01 and the regression that yielded the highest explanatory power (i.e. R^2) was kept. This was done on the entire data-set (i.e. 100 firms) as well as on the subset of partnerships that had respectively less than 50 partners (i.e. 54 small partnerships with less than 500 employees) and more than 50 partners (i.e. 46 large partnerships (very large firms) with more than 500 employees).

Results (summarized in the table (1)) show that taking into account partner's productivity fluctuations into the proposed approach increases the explanatory power of the proposed model on the

Parameter	Estimates across all partnerships	Estimates across small partnerships	Estimates across large partnerships
ϕ	-0.048	0.078	-0.11
α_0	0.14 (0.01)	0.22 (0.01)	0.11 (0.01)
$\alpha_0.\gamma$	-0.0005 (0.0003)	0.0002 (0.0006)	0.0007 (0.0003)
R^2	97%	97.1%	98%

Table 1: 2018 US accounting partners revenue sensitivity analysis.

data-set (R^2 grows by 1 to 2%). This also highlights significant differences across small and large accounting partnerships. First, in small partnerships, partners' productivity appears to decrease as the partnership expands ($\phi \geq 0$), while it increases in large partnerships (see figure (2)). Second, adding an employee in a small partnership generates twice as much additional revenue than in a large one. Third, coordination does appear to be negligible in small partnerships while it becomes a significant topic in large ones [5].
Sections (2) and (3) have shown that partners revenue (R) is dependent in both the amount of employees that ensure the service production (L) and the partnership size (S). Those two variables are yet not independent: partners indeed seek to maximize the firm profitability (which drives their earnings) and have to adjust their production structure as the partnership evolves (i.e. L is a function of S). The associated performance optimal trajectory will now be discussed throughout sections (4) to (6).

4 PSFs organic growth.

As seen in sections (2) and (3), partners' revenue (R) changes with both the size of the partnership (S) and the number of employees (L) they leverage. As partners are compensated based on their profit Π, while their employees perceive a fixed wage c (so that $\Pi(S,L) = R(S,L) - c.L$), partners have to adjust their leverage so as to maximize their earnings as their partnership is scaling up (i.e. L is a function of S). This evolution is made of two components. On one hand, partnerships grow organically as employees absorb enough knowledge over time to become able to sell services and be promoted to partners. On the other hand, partnerships can grow inorganically as their constituents search and recruit new peers externally.
Organic growth has traditionally been considered as the main strategic option for PSFs. The associated promotion opportunities indeed act both as a long term incentive for employees (see [Lazear and Rosen, 1981] for a discussion on tournaments incentives) and as a way for the firm to increase its revenue (and possibly its employment) by identifying individuals with potential (see [Maister, 2012], [Maister, 1982]). Meanwhile, inorganic growth remains uncommon (see [Morris and Pinnington, 1998]), because it is expensive and risky (see [Farhadi, 2007] for a discussion on accounting firms). As such, this section will explain how partners can optimize their earnings in a PSFs that grows organically while section (5) will detail the changes entailed by the use of inorganic growth mediums.
PSFs organic growth has been thoroughly discussed in the literature. The most prominent school of thoughts on the subject (see [Galanter and Palay, 1994] for a review pertaining to law firms) stresses

[5] Interestingly this means that every time that an employee has to coordinate with another, it takes about 0.2% of their time. As a point of reference, this is of a same order of magnitude as the coordination time observed in societies. This was assessed and reported by [Dunbar, 1993] to take about 0.3% of an individual time.

that it obeys a tournament structure, where a fixed portion θ of employees make it to partner [i.e. $dS_t = \theta.E_t.dt = \theta.S_t.L(S_t).dt$]. For partners to maximize their earnings over a period T, they must therefore find a delegation strategy ($L(.)$) that solves the following problem:

$$\Pi = \max_{L(.)} \int_{t=0}^{T} [R(L(S_t), S_t) - c.L(S_t)].dt \leftrightarrow \partial_t \Pi + \max_{L(.)} (\theta.S.L.\partial_S \Pi + R(S, L) - c.L) = 0 \quad (7)$$

This delegation strategy depends in the service delivery structure of the firm and notably changes if individual coordination (γ) is important as seen in the following proposition (proof can be found in the appendix) [6].

Proposition 4.1 *When individual coordination is not negligible (i.e. $\gamma \neq 0$), partners' optimal delegation strategy ($L(.)$) depends in their productivity (α):*

$$L(S_t) = \sqrt{\frac{1 + \gamma.L(S_0)^2}{\gamma} . \frac{\alpha(S_0)}{\alpha(S)} - \frac{1}{\gamma}} \quad (8)$$

If partners' productivity decreases as the partnership expands ($\partial_S \alpha \leq 0$), the firm's partnership size increases organically in an "exponential" fashion (i.e. has a increasing growth speed over time). Otherwise, the partnership grows but exhibits a decreasing organic growth rate.
If individual coordination can be neglected (i.e. $\gamma = 0$), the optimal delegation strategy is independent of the service price and depends only in the firm delivery speed ($T_{prod}(S_t)$):

$$L(S_t) = L_{max}(S_t) = (\frac{T_{prod}(S_t)}{T_{sales}}) \quad (9)$$

The partnership organic growth speed then increases only if the service production activities slow down as the partnership expands (i.e. $\partial_S T_{prod} \leq 0$)

There are two notable features of the model developed through sections (2) to (4). First increasing PSFs' human capital (i.e. expanding the partnership) does not necessarily translate in growth. This highlights PSFs specificity in the industrial organization landscape as most of the literature considers firm growth as synonym of employment. Second the model shows that to achieve organic growth means speeding up service delivery (i.e. to standardize/ commoditize), whether or not delegation exhibits decreasing return to scale (for instance because of coordination). This is interesting because it shows that setting up a PSF as a partnership may not always be a suitable answer as the human capital accumulation does not necessarily entails the desired returns. Although this discussion is not in scope of this paper, this could constitute an interesting avenue for further research.

Example. The proposed model can be used with the 2018 US *Public Accounting report* to infer some properties on US accounting firms, assuming of course organic growth as the main medium of partnership expansion. First, employees promotion chances can be estimated as the data-set records both the growth rate of each firm's partnership ($\frac{\partial_t S}{S} \in [-17.5\%; +50\%]$ with an average of 7.4% (11.5%)) and their leverage. This can be done directly via OLS (as $\frac{\partial_t S}{S} = \theta.L$) and results in $\theta = 0.52\%$ (0.14%),

[6] The accounting example developed in section (3) has indeed shown that labor coordination only occurs in certain specific cases (e.g. accounting partnerships with more than 50 partners).

which is on par with the literature [7] .
Second, the data-set can be used to infer how partnerships production speed is evolving. In the case of small [pres. large] partnership (i.e. less [reps. more] than 50 partners), leverage is negatively [resp. positively] correlated with the partnership size. Proposition (4.1) therefore entails that delivery speed increases [resp. decreases] as small [resp. large] accounting partnerships. This however appears counter factual in the case of large partnerships. As the market is subject to firms consolidation (see US census statistics) to gain in competitiveness, one would expect that large partnerships keep on improving their service delivery model while expanding.

In summary, the performance model developed until section (4) is the reflect of the current knowledge available in the literature and provides a performance optimal development strategy for partnerships. However, its core assumption is that partnerships grow organically which, as shown in the previous example, may lead to counter factual results. The next section will therefore expand the model to cover partnerships dynamics in a more holistic fashion and explain the observed revenue and employment patterns.

5 Profit-optimal PSFs growth strategies.

Although organic growth has been the main focus of the management and economic literature pertaining to PSFs performance, it is a slow process[8]. To speed up their development and maximize partners earnings over a given time horizon, PSFs have the opportunity to compete in the labor market to source/attract partners from their competitors. The expertise of each partner is indeed general rather than firm specific. As such, on top of their organic growth patterns, PSFs bring in $H \geq 0$ new partners and also loose $T \geq 0$ partners to other firms. Their partnerships' evolution dynamics thus become:

$$dS = (\overbrace{\theta.S.L(S)}^{\text{Organic growth}} + \overbrace{H(S)}^{\text{External hire(s)}} - \overbrace{T(S)}^{\text{Competition pressure}}).dt \tag{10}$$

This search for new partners represents a significant investment, which is usually shared equally by all the current partners (see [Groysberg et al., 2004] for instance). It is therefore subject to an optimization as the associated costs are empirically known to be super linear (see [Christensen et al., 2005]) (i.e. the unit cost of bringing a new partner in increases with the total of amount of partners externally hired). In the literature, those costs are commonly assumed to be quadratic (see [Burdett, 1978] for instance) (i.e. $c_H.H^2$). To maximize their earnings, partners have therefore to find a strategy ($L(.)$,$H(.)$) which solves an enhanced version of the problem (7) defined in section (4):

$$\Pi = \max_{L(.),H(.)} \int_{t=0}^{T} [(p.N)(S_t) - c.L(S_t) - c_H.\frac{H(S_t)^2}{S_t}].dt \tag{11}$$

As seen in section (4), the overall growth strategy of a partnership is dependent in the nature of its service delivery model and the partners earnings maximization leads to the following proposition (please refer to the appendix for a proof).

[7]For instance, [Galanter and Palay, 1990] reports that it takes about 10 years for an individual to get a 5 to 20% change to get promoted, which means that promotion rates (θ) are in the [0.2%; 0.5%] range

[8]With promotion rates θ averaging [0.2%,0.5%] per year and a leverage per partner of 10 employees, small partnerships (i.e. with less than 50 partners) only promote about 1 new partner per year

Proposition 5.1 *When service delivery is not subject to frictions due to coordination (i.e.* $T_{coord} = 0$*), partners maximize their earnings by delegating all of their production activities to employees (i.e.* $L = L_{max}(S) = (\frac{T_{prod}}{T_{sales}})(S)$*) and seeking peers externally at a rate* $H(.)$ *given by* [9]*:*

$$H(S_t) = T(S_t) - \theta.L_{max}.S_t + \sqrt{(T(S_t) - \theta.L_{max}.S_t)^2 - \frac{S_t}{c_H}.((\alpha - c).(L_{max} + 1) - \bar{C}))} \tag{12}$$

In a competitive landscape, the external search therefore secures the partnership growth (i.e.$\partial_t S \geq 0$) in the absence of labor coordination (e.g. the growth of small US accounting partnerships according to the previous examples). However, when coordination occurs, the optimal partnership growth strategy entailed by problem (eq. (11)) becomes more complex.

Proposition 5.2 *When the delivery of professional services requires significant coordination (i.e.* $\gamma \neq 0$*), if the optimal external search speed* ($H(.)$) *of a partnership grows with partner productivity and leverage, it becomes asymptotically negligible as the partnership expands (i.e.* $H(S)/S \rightarrow 0$*):*

$$H = \frac{\alpha(S).(2.\gamma.L - (1 + \gamma)) + c}{2.c_H.\theta} \tag{13}$$

The delegation strategy of partners however grows more complex and differs from the partnership organic patterns as delegation and promotion potentially becomes a way to mitigate the partner market competitive pressure (i.e. $c - \theta.c_H.2.T(S)$*)* [10] *:*

$$L(S_t) = \frac{-(c - \theta.c_H.2.T(S_t)) \pm \sqrt{(c - \theta.c_H.2.T(S_t))^2 - 4.(\theta^2.c_H.S_t + \alpha(S_t).\gamma).M(S_t)}}{\theta^2.c_H.S_t + \alpha(S_t).\gamma} \tag{14}$$

Looking back at section (3), the case of US accounting partnerships can be used to illustrate the proposed theory on PSFs profitable growth.

Example. In small US accounting partnerships (i.e. firms with less than 50 partners) coordination is not significant. Therefore, partners productivity ($\alpha(S)$)[11] and delegation strategy ($L_{max}(S)$)[12] evolution with the partnership size can be directly estimated from the data-set. Assuming that the cost of an employee in the US is $c = 0.06k\$$ per year (as per the US census), that partners turnover rate (i.e. $T(S)/S$) is proportional to their productivity (see [Jovanovic, 1979]) and worth 10% on average, that search costs (c_H) for new partners are worth 1M$ ([Groysberg et al., 2004]), the proposed theory predicts that the growth rate of US accounting firms is stable (see figure (3)). This is aligned with the data set (i.e. there was no correlations between small partnership size and their growth rates) and comes with two main observations. First, external hiring appears to be responsible for most of the growth of those small US accounting partnerships. Second, as those partnerships expand, partners turnover increases and so does external hiring as a compensation mechanism.

[9] Up to a constant $\bar{C} = \frac{c_H.H_0}{S_0}.(H_0 + 2.\theta.S_0.L_{max}(S_0) - 2.T(S_0)) + (\alpha(S_0) - c).(L_{max}(S_0) + 1)$

[10] With $M(S) = (\bar{C}.\frac{\theta^2.c_H.S}{\gamma.\alpha} + \frac{\theta.c_H}{\gamma}.(\theta.S - T.(1 + \gamma)) + \frac{c^2+\alpha^2.(1+\gamma)^2}{4.\gamma.\alpha} - \frac{3.c.(1+\gamma)}{2.\gamma})$ and $\bar{C} = -\alpha(S_0).(\gamma.L(S_0)^2 + 1 - \frac{T(S_0)}{S_0.\theta}.(2.\gamma.L(S_0) - (1 + \gamma))) - \frac{(\alpha(S_0).(2.\gamma.L(S_0)-(1+\gamma))+c)^2}{4.c_H.\theta^2.S_0}$

[11] Running the regression $\alpha = a + b.S$ yields that partner's productivity is worth $a = 0.19k\$$ (0.012) per year and decreases by $b = -480$ \$ per year (380) for every new partner that is added to the firm structure.

[12] The leverage sensitivity to the partnership size can be estimated with the regression $L_{max} = a + b.S$. An OLS estimation on the US accounting data-set yields that $a = 22.86$ (1.91) and for every new partner the leverage decreases by $b = -0.34$ employee (0.060).

In large US accounting partnerships (i.e. more than 50 partners), the dynamics are yet different. As labor needs to be coordinated when firms are large (see section (3)), the proposition (5.2) stresses that in large US accounting partnerships, external partner sourcing rapidly decreases to becomes negligible, a property that is empirically supported by the data-set (see figure (4)).

In summary, this section has shown that, in a competitive environment, if external search can be used to improve profitability, it only ensures the growth of nimble partnerships (for instance small US accounting ones) and not the growth of less agile ones. This duality in PSFs has already been empirically observed, although, to my knowledge, never explained. For instance, [Galanter and Palay, 1990] reported a difference in growth modes in law firms as they grew bigger and broader evidences in the field of industrial organization (see [Evans, 1987]) have also stressed that firm growth is dependent in its maturity.
The proposed theory however does not explain how service delivery speed (T_{prod}) improves as the partnerships expands. Although the topic has been usually approached in the management literature under the lens of process improvements (see [Løwendahl, 2005]), recent findings ([Frey and Osborne, 2017]) suggest that technology has the potential to replace labor at scale. Those type of investments could impact the proposed performance paradigm, a topic which will be the focus of the following section.

6 Technology and PSFs performance: a discussion

The sections (2) to (5) have demonstrated how PSFs can maximize their profitability in a market where competition is productivity-driven. This was notably shown to be synonym of growth when service delivery does not require coordination. Although the proposed framework closely aligns with the current literature related to PSFs ([Maister, 2012]), recent findings (see [Arntz et al., 2016]) have stressed that in upcoming years, 15% to 25% of PSFs labor could be replaced by technology.
This could change the competitive equilibrium of the sector in the future as partners can invest an amount I in technology to speed up delivery[13]. Assuming that technology becomes the main lever to pull to lower production time (i.e. $T_{prod}(I)$ with $\partial_I T_{prod}(I) \leq 0$) means that partners' productivity (α) and maximum leverage (L_{max}) become a function of technology. For partners to maximize their earnings, they must therefore find an optimal strategy around the delegation of their production tasks ($L(.)$), their peers external search ($H(.)$) and their accumulated technology investments ($I(.)$). Assuming that technology investments are shared equally within the partnership, the work done in section (3) to (5) entails that partners' optimal strategy is defined as a solution of:

$$\partial_t \Pi + \max_{L(.),I(.),H(.)} ((\theta.S.L + H - T)\partial_S \Pi + \alpha(I).\frac{(1 + L.(1+\gamma) - \gamma.L^2)}{1 + (\frac{T_{prod}}{T_{sales}})(I)} - c.L - \frac{(c_H.H^2 + \partial_t I)}{S}) = 0 \quad (15)$$

As in previous sections, several cases appear depending in the need for coordination while delivering services (note that proves can be found in the appendix.)

Proposition 6.1 *If labor coordination can be neglected (i.e. $T_{coord} = 0$) and technology is an available option to become more competitive, partners investments are driven by the following optimal program*

[13] Note that this extends the classical literature around firm growth and technology investments (see [Lucas and Prescott, 1971]) by providing a specific application to professional services firms.

(with $\bar{C}$ and $\bar{p}$ constants):

$$\partial_S I = 2.\sqrt{c_H.S.(\frac{V.\tau}{T_{sales}} - \bar{p} + \bar{C}) + c_H.(c+\bar{p}).(\frac{(c+\bar{p})}{4.c_H.\theta^2} - \frac{T}{2.\theta})} \quad (16)$$

They then delegate as much work as possible (i.e. $L = L_{max}(I)$) and their optimal external search effort (H) becomes a function of their accumulated technology investments ($H = \frac{c+\bar{p}}{2.c_H.\theta} + \frac{\partial_S I}{2.c_H}$).

Technology therefore entails two main changes to the strategies of nimble partnerships (i.e. partnerships where labor does not require coordination). First, when technology is available, nimble PSFs profit maximization is no longer synonym of partnership growth. Second, technology leads to reduction of organic growth opportunities and an increased reliance on external searches.
When employees coordination ($T_{coord} \neq 0$) can no longer be neglected, the effect of technology on PSFs performance depends in the magnitude of the disruption.

Lemma 6.2 *If technology represents a minor disruption (i.e. if there is a level of investment in technology I^* which minimizes the production time (i.e. $\partial_I T_{prod}(I^*) = 0$) and if the maximum leverage post investment ($L_{max}(I^*)$) is still superior to the current leverage ($L(S_0)$)), then the optimal investment strategy for partners is to perform a single up-front investment (i.e. $\forall S \geq S_0$, $I(S) = I^*$) and the optimal delegation and external hiring strategies ($L(.)$ and $H(.)$) still follow the patterns described in section (4).*

Proposition 6.3 *However if technology is disruptive (i.e leads to a drastic reduction in the delegation needs), partners profit maximization entails maximizing delegation ($L = L_{max}(I)$). Technology investments are then given by (with $R(I) = \alpha(I).(1 + (1+\gamma).L_{max}(I) - \gamma.L_{max}(I)^2)$ and $\bar{C}$ a constant):*

$$\partial_S I = \sqrt{4.c_H.S.\bar{C} + \frac{(\partial_I R - c.\partial_I L_{max})^2}{\theta^2.(\partial_I L)^2} + 4.S.c_H.(R(I) - \frac{T.c}{\theta.S} - (\frac{\theta.S.L_{max} - T}{\theta.S}).(\frac{\partial_I R}{\partial_I L_{max}}))} \quad (17)$$

Finally external search activities are adjusted to account for both technology investments and the leverage structure ($H(S) = \frac{\partial_S I}{2.c_H} + \frac{c}{2.\theta.c_H} - \frac{\partial_I[(1+(1+\gamma).L_{max}(I)-\gamma.L_{max}(I)^2).\alpha(I)]}{2.c_H.\theta.\partial_I L_{max}}$).

Example. Looking back at the 2018 US *Public Accounting report*, the analysis performed in sections (4) and (5) has shown that partners' productivity in small accounting partnerships (i.e. less than 50 partners) was decreasing as the partnership expands. The proposed theory therefore suggests that technology is not an available option for those type of firms. In larger partnerships, partners' productivity has been shown to increase, which is compatible with technology investments. However in the absence of more granular data, it not possible to further discuss to which extent does technology currently disrupt those firms.
In summary, technology has the potential to change the way PSFs seek to improve their performance. When coordination can be neglected, this happens through a modification of their external search behavior. Otherwise, technology enable firms to over-come the limits entailed by labor coordination. As technology starts to be more massively adopted in those practices, it would be interesting to witness how performance patterns change.

7 PSFs growth heterogeneity sources: a discussion

The model developed in this paper through section (2) to (5) has explained how PSFs currently maximize their performance. The associated examples have yet shown that proposed theory does not capture all of the PSFs heterogeneity in performance. This section therefore offers a few additional insights on why the model could yield different performance outcomes for different firms.

Service nature & PSFs performance. The professional services typology of [Løwendahl, 2005] was used in this paper to model a standard professional services delivery infrastructure (i.e. θ, T_{sales}, T_{prod}, γ) and to derive an optimal performance strategy. However as the field covers a broad range of industrial sub-sectors (e.g. the legal, accounting, engineering ()...) services), each of those dimensions can heavily vary from one service to the other. According to [Maister, 2012] (which framework is summarized in 5), those differences lead to 3 services types: "brains" (type "B"), "gray hairs" (type "G") and "procedure" (type "P"). "Brains" are client relation-based services, offered to long-term clients. "Grey hairs" (type "G") are more creative problem solving services , which leverage an expertise in a given field to solve the most unique and difficult problems there are. Finally "Procedure" (type "P") services are offer "off the shelf" (i.e. commoditized).
Those differences can be used to further explain some differences amongst PSFs. First the more complex the problem at the heart of the service, the more time an individual requires to develop the required selling skills ([Løwendahl, 2005]). Therefore the probability of a producer to become a seller is lower in brain (resp. gray hair) firms than in gray hair (resp. procedure) professional services (i.e. θ decreases). Second, the more complex the service, the lower the ratio of production over sales activities (i.e. L_{max} decreases) and the more difficult it becomes to invest in technology (i.e. $|\partial_I T_{prod}|$ decreases). Leveraging the superscript "B"/"G"/"P" to refer to the service type and the notations of the previous sections, this translates into the following service delivery model properties.

Lemma 7.1 *The more complex the service, the slower its organic growth (i.e. brains [resp. gray hair] services promote less partners than gray hair [resp. procedure] ones).*

$$\forall S > 0 \qquad (\theta.L)_B(S) \leq (\theta.L)_G(S) \leq (\theta.L)_P(S)$$

This means that to grow at a set rate, more complex services must rely more heavily on external search than commoditized ones. Additionally, as PSFs grow, they diversify their activities to deliver on a portfolio of service (see [Greenwood et al., 2005] for a discussion and the US accounting report for empirical data on the mix of services accounting firms offer for an example). Their overall performance can therefore differ based on their portfolio mix.
If known differences in services characteristics can be used to explain some of the observed heterogeneity in PSFs performance, markets, defined in terms of clients and competitors pools can be extremely different from one place to the next, which naturally calls for a extension of the discussion on performance heterogeneity.

Markets & PSFs performance. From a market standpoint, the development of a professional service occurs in 4 steps ([Carman and Langeard, 1980]) through a mix of a customer segment and geography related activities. The first step in growth consists in competing to attract more and more clients in a given customer segment after a local introduction. The second step is about developing an offering by complementing the existing service in a peripheral fashion through sub-services that can increase competitiveness by boosting the overall productivity ([Skaggs and Youndt, 2004]). The

third step relies on the service introduction in a new geography. The last step is about expanding the service to a new customer segment, which often leads to its redefinition.
The choice of a given market as defined by its geography (for example choosing the market of the technology firms with more than 200 employees in London - United Kingdom versus the one in Berlin - Germany) has been empirically shown to drive growth differences. PSFs that introduce their services in large and fast growing metropolitan areas indeed experience a quicker growth than their counterparts ([HUallacháin and Reid, 1991], [Bryson et al., 1993]). Looking back at the framework developed in section (3), the growth of a service is indeed capped by the overall market evolution as:

$$\underbrace{\int_{S_{min}}^{S_{max}} N(s)g(s)ds}_{\text{Number of clients that can be served by partners}} \leq \underbrace{\int f(z)dz}_{\text{Total number of clients available in the market}} \tag{18}$$

Therefore the location choice of a PSF can boost (resp. reduce) its growth opportunities if the market grows quickly (resp. is or becomes saturated) (i.e. the constraint (18) becomes binding). Note that the choice of the service extension to another customer segment has the equivalent effect as it relaxes the condition (18) for the firm. Therefore PSFs generally both expand geographically and across customer segment at the same time in order to maximize their growth potential ([Dhandapani and Upadhyayula, 2015]). Client demand therefore drives partnership organic growth rates (i.e. $\theta = \frac{\int \partial_t f(t,z)dz}{\int L(s)g(s)ds}$).

Example. To illustrate those differences, let's assume that a given management consulting service is introduced by a firm in 2016. This service is offered to legal and accounting firms in France that have between 50 and 250 employees. This represents a client pool $N(t)$ of 340 firms according the OECD Structural Business Statistics (ISIC Rev. 4) database. The same database reports that the segment grows by about 18 firms on average per year. For the sake of simplicity, all the firms that supply this service are assumed to have the same leverage ratio $L_{max} = 5$ (which is on par with the benchmarks observed in the literature [Kipping and Kirkpatrick, 2013]). Let us also assume that a partner can sell services to 8 to 10 clients when production is fully delegated (see ([Zerni, 2012]) for benchmarks). This means that the market has room for 85 partners and can add an extra 2 new partners per year, which result in an overall promotion rate of $\theta = 1\%$.

Partners characteristics & PSFs performance. Outside of markets and service nature, partners characteristics are also known to generate heterogeneous PSFs performance patterns. First partners' productivity (i.e. α) fluctuates due to random exogenous shocks (see [Lucas and Prescott, 1971]). This generates "noise" with respect to the overall performance signal of their partnership, notably in firms which services portfolio is not extensive. Second, the choices made by partners in terms of external search have also an impact on partnerships' performance. When PSFs engage in larger corporate acquisitions in order to extend their partnership, the amount of change that the operation entails has notable consequences on PSFs' growth rates (see [McKelvie and Wiklund, 2010]). Acquisitions indeed entail lay offs as efficiencies of scope are identified during the merger/acquisition process (see [McKelvie et al., 2006]) and reduces revenue growth speed ([Coad, 2007]).Finally, the ambitions of constituent partners have of course an impact on the firm performance ([McKelvie and Wiklund, 2010]). Empirical work has indeed stressed that some leaders are not interested in maximizing business performance ([Wiklund et al., 2003]) as they focus on non financial questions such as the well-being of their employees or their independence ([Quader, 2007]) and level of control in their partnership. Note that even for partners with a focus on financial objectives, preferences for short term over long term profitability can lead to different decisions.

8 Conclusion.

Building upon a picture of professional services production mechanisms inspired by the seminal work of ([Løwendahl, 2005]) and ([Maister, 2012]), this paper provides performance optimal management policies for PSFs in terms of labor delegation, external partner search and technology investments. The associated theory has been applied to explain the recent growth patterns of US accounting firms. Two new streams of research now naturally emerge from the proposed theory. First, a more granular perspective on the development of PSFs in terms of markets/location strategy appears of interest (see ([Bodenman, 2000],[Harrington, 1995]) for empirical discussion elements). It would indeed prove useful to develop a business development as well labor cost optimization blueprint for PSFs considering an increasingly globalized service market (see [Jensen et al., 2005] for a discussion on the impact of trade on professional services). This would notably expands on the high level work around competition presented in sections (3) and (7) of this paper. Second, it would be interesting to understand at a more granular level the incentive mechanisms at play in professional service firms. This would indeed provide a better idea of what cements such firms service delivery model and would potentially help understand why most PSFs start small, do not grow and die ([Storey, 2016]).

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9 Appendix.

9.1 Proofs.

Lemma 9.1 *If $\tau > T_{sales} + T_{prod}$, a partner is able generate revenue on its own. Otherwise, for the service delivery to be possible, the following condition must be met $\frac{1}{2.T_{coord}}(\tau + \frac{T_{coord}}{2})^2 \geq (T_{prod} + T_{sales} - \tau)$ and a partner requires a minimum of $\underline{L} = (\frac{\tau}{T_{coord}} + \frac{1}{2}).(1 - \sqrt{1 - \frac{4.(T_{prod}+T_{sales}-\tau)}{2.\tau+T_{coord}}})$ employees.*

Proof. If a partner doesn't have enough bandwidth to service a client on its own (i.e. $\tau < T_{sales} + T_{prod}$), he/she needs to delegate the service production to a number of employees $\underline{L} > 0$ solution of the following equation:

$$L.(\tau + \frac{T_{coord}}{2}) - L^2.\frac{T_{coord}}{2} = (T_{prod} + T_{sales} - \tau)$$

For this solution to exist, the following condition must be met $\frac{1}{2.T_{coord}}(\tau + \frac{T_{coord}}{2})^2 \geq (T_{prod} + T_{sales} - \tau)$, which lead to the proposed solution.

Lemma 9.2 *A partner can not delegate more production activities than available (i.e $N_{prod}^{delegated} \leq N$). If $\frac{1}{2.T_{coord}}(\tau + \frac{T_{coord}}{2})^2 \leq \frac{\tau.T_{prod}}{T_{sales}}$, there is no restriction on the maximum number of employees that a partner can leverage. Otherwise, the maximum leverage L_{max} is $(\frac{\tau}{T_{coord}}+\frac{1}{2}).(1-\sqrt{1-\frac{4.\tau.T_{prod}}{T_{sales}.(2.\tau+T_{coord})}})$*

Proof. The condition $N_{prod}^{delegated} \leq N$ can be translated into the following condition on leverage:$L.(\tau + \frac{T_{coord}}{2}) - L^2.\frac{T_{coord}}{2} \leq \frac{\tau.T_{prod}}{T_{sales}}$. This means that, if it exists, the maximum number of employee a partner can leverage is solution of:

$$L.(\tau + \frac{T_{coord}}{2}) - L^2.\frac{T_{coord}}{2} = \frac{\tau.T_{prod}}{T_{sales}}$$

This leads to the proposed solution and condition.

Proof of proposition (3.1) For clients to have an interest in purchasing the service, prices must be such that $p(S) \leq V$. Additionally the market productivity maximization (eq.(4)) leads to:

$$\forall S > 0 \qquad \partial_S p = -\frac{\overbrace{[V - p(S)]}^{\geq 0}}{\underbrace{T_D(S)}_{\geq 0}}.\partial_S T_D(S) \qquad \partial_S \alpha = \tau.\frac{\partial_S p.T_D - \partial_S T_D.p}{T_D^2} \tag{19}$$

Proof of proposition (4.1) As $pN(L,S) = \alpha(S).(1+(1+\gamma).L-\gamma.L^2)$, the maximization problem $[\max_{L(.)}]$ in equation (7) leads to: $\frac{2.\gamma.L.\alpha(S)+c-\alpha(S).(1+\gamma)}{\theta.S} = \partial_S\Pi$. This means that the overall equation (7) translates into: $\partial_t\Pi + L^2.\gamma.\alpha(S)+\alpha(S)) = 0$. As $\partial_{tS}\Pi = 0$, this can be differentiated against S to yield: $\frac{\partial_s\alpha}{\alpha} = -\frac{2.L.\gamma}{1+\gamma.L^2}.\partial_S L$, which then leads to eq. (8). If coordination can be neglected (i.e. $\gamma = 0$), the partner profit maximization problem (eq. 7) leads to $L = L_{max} = \frac{T_{prod}}{T_{sales}}$. The problem (eq. 7) indeed becomes linear in L and doesn't admit a strict maximum. This forces L to be set at L_{max}.
As partners define their optimal delegation scheme, the pace at which the partnership evolves becomes set. When coordination occurs, the partnership grows organically at rate of $\theta.L$, therefore as the leverage increases, so does the partnership growth speed $(\partial_t(\frac{\partial_t S}{S}) = \partial_t S.\theta.\partial_S L = -\frac{\theta}{2.L}.\frac{1+\gamma.L_0^2}{\gamma}.\frac{\alpha_0.\partial_S\alpha}{\alpha(S)})$.

Proof of proposition (5.1) When coordination effects are neglected, profit maximization entails maximizing delegation. In this case, the condition $\partial_S\Pi = 2.c_H.\frac{H}{S}$ holds and the problem becomes: $\partial_t\Pi + \frac{c_H.H}{S}.(H+2.\theta.S.L_{max}-2.T(S))+(\alpha-c).L_{max}+\alpha = 0$. When differentiating against S, it comes that:$[\frac{c_H.H}{S}.(H+2.\theta.S.L_{max}-2.T(S)) - \frac{c_H.H_0}{S_0}.(H_0+2.\theta.S_0.L_{max,0}-2.T(S_0))] = [(\alpha_0-c).L_{max,0} - (\alpha_t - c).L_{max,t}+\alpha(S_0)-\alpha(S_t)]$, which leads to the desired result.

Proof of proposition (5.2) **Proof**. The problem (11) is equivalent to: $\partial_t\Pi + \max_{L(.),H(.)}((\theta.S.L + H - T).\partial_S\Pi + p.N(L) - c.L - c_H.\frac{H^2}{S}) = 0$. The maximization against H and against L yields that $\partial_S\pi = \frac{2.c_H.H}{S} = \frac{\alpha(S).(2.\gamma.L-(1+\gamma))+c}{\theta.S}$, which leads to the proposed solution for external hires (see eq. (13)). The problem (eq.11) then becomes: $\partial_t\Pi + (\gamma.L^2+1-\frac{T}{S.\theta}.(2.\gamma.L-(1+\gamma))).\alpha(S)+\frac{(\alpha(S).(2.\gamma.L-(1+\gamma))+c)^2}{4.\theta^2.c_H.S} = 0$. Differentiating against S and leveraging the fact that $\partial_{tS}\Pi = 0$, the proposed result appears as: $\partial_S((\gamma.L^2+1-\frac{T}{S.\theta}.(2.\gamma.L-(1+\gamma))).\alpha) = -\partial_S(\frac{(\alpha(S).(2.\gamma.L-(1+\gamma))+c)^2}{4.\theta^2.c_H.S})$.

Proof of proposition (6.1) As seen in the previous sections, the absence of coordination leads to $L = L_{max}(I)$ and the profit maximization problem becomes: $\partial_t\Pi + \max_{I(.),H(.)}((\theta.S.L_{max}(I) + H - T)\partial_S\Pi + \frac{p(I).\tau}{T_{sales}} - c.L_{max}(I) - \frac{(c_H.H^2+\partial_t I)}{S}) = 0$. As $\partial_t I = \partial_S I.(\theta.S.L + H - T)$, differentiating against H and I yields: $\partial_S\Pi = \frac{2.c_H.H}{S} = \frac{c+\bar{p}}{\theta.S} + \frac{\partial_S I}{S}$. This leads to the desired formula for $H(.)$ and $I(.)$.

Proof of proposition (6.2) When coordination is significant, partners profitability maximization leads to : $\partial_t\Pi + \max_{L(.),I(.),H(.)}((\theta.S.L + H - T)\partial_S\Pi + \alpha(I).(1 + L.(1+\gamma) - \gamma.L^2) - c.L - (\theta.L + \frac{H-T}{S}).\partial_S I) - \frac{c_H.H^2}{S} = 0$. Differentiating against I and assuming that delegation is $L < L_{max}(I)$, the condition: $\partial_I\alpha.(1 + L.(1+\gamma) - \gamma.L^2) = 0$ appears, which leads to the suggested result.

9.2 Figures.

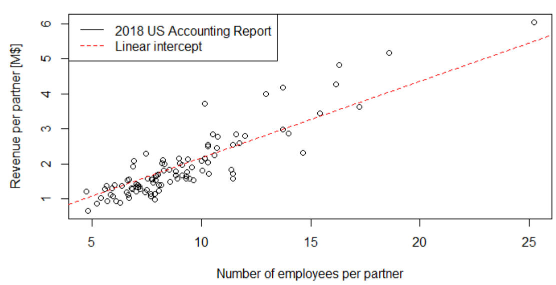

Figure 1: Revenue evolution with delegation in US accounting firms.

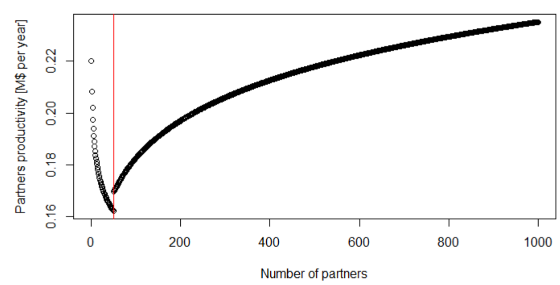

Figure 2: Partner productivity evolution with partnership size in US accounting firms.

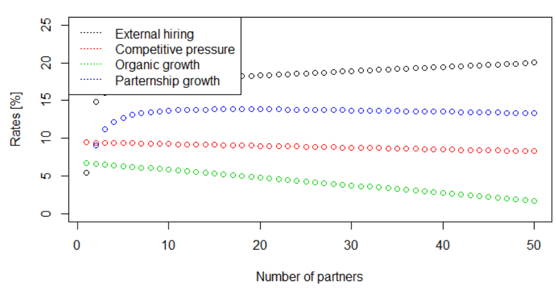

Figure 3: Partnership growth rates evolution in small US accounting partnerships (i.e less than 50 partners).

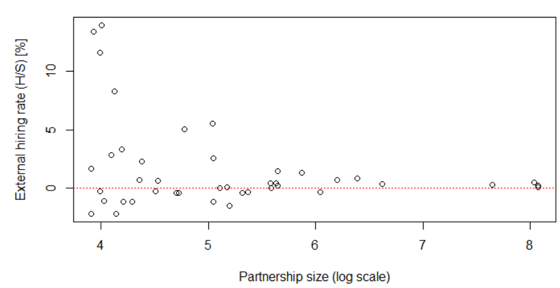

Figure 4: Partnership estimated external hiring rates evolution in large US accounting partnerships (i.e more than 50 partners). g

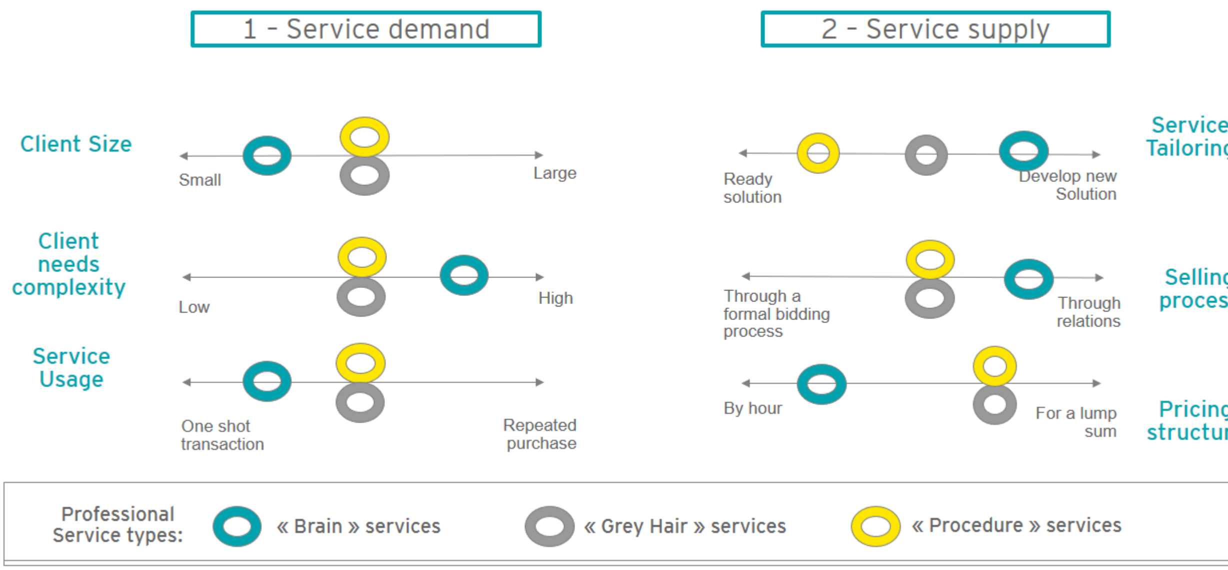

Figure 5: Main professional services types.