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Extended linear formulation of the pump scheduling problem in water distribution networks

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ABSTRACT

This paper presents a generic non-compact linear programming approximation of the pump scheduling problem in drinking water distribution networks. Instead of relying on the binary on/off status of the pumps, the model draws on the continuous duration of activation of pump combinations, whose entire set is computed in a preprocessing step by ignoring the pressure variation in the water tanks. Preprocessing is accelerated using network partition and symmetry arguments. A combinatorial Benders decomposition-based local search takes the approximated solution as input to derive a feasible solution. Our experiments on two different benchmark sets, with fixed- or variable-speed pumps, show the accuracy of the approximated formulation and the ability of the matheuristic to compute near-optimal solutions in seconds, where concurrent, more specialized approaches need minutes or hours.

1 INTRODUCTION

With the evolution of the power sector – because dynamic pricing is a savings opportunity for water network operators [6] – together with advances in mixed-integer nonlinear programming (MINLP), recent years have seen a renewed interest in minimizing the pumping costs in drinking water distribution networks.

The so-called *pump scheduling problem* is a hard combinatorial non-convex optimization problem. A variety of solution approaches have been investigated, but they often inefficiently deal with large or medium networks, and many small instances are still open. A first category of approaches (e.g. [6, 7, 10]) combine a numerical simulator, to compute the feasible hydraulic balances, with an exact or heuristic optimization algorithm, to schedule the pump operations at minimum cost. Separating feasibility from optimization makes the convergence of these approaches slow, resulting in sub-optimal solutions. A second category of approaches formulate the whole problem as a MINLP with simplified hydraulic constraints, based either on piecewise-linear approximations (e.g. [8, 9]) or convex relaxations [2, 3, 13]. These approaches only apply to small networks, because of the combinatorial nature of the models, and they may return impracticable solutions. Instead, Burgschweiger et al. [4] keep the non-convex constraints in their model but relax the binary on/off pump activation variables by aggregating them. This relaxation is only suitable to large city-wide networks where dozen of pumps are installed in parallel in each pumping station.

In this paper, we are interested in tackling intermediate-size networks with a mixed approach. We propose to approximate the MINLP model by decoupling feasibility and optimization in the way of Dantzig-Wolfe decomposition: by ignoring the pressure variation in the water tanks, we can compute the feasible hydraulic balances for all pump configurations as a preprocessing step, then derive a non-compact linear programming (LP) model based on the durations of activation of these configurations. We apply network partition and symmetry arguments to accelerate the preprocessing without hindering optimality. While the approximated LP solutions may be accurate enough to be practically implemented in pump controllers, we also propose to derive feasible solutions for the original MINLP with a local search approach, adapted from the combinatorial Benders decomposition of Naoum-Sawaya et al. [10]. Finally, we generalize the approach to networks with variable-speed pumps or pressure-reducing valves, which are often overlooked in the literature.

Experiments on the Poormond [6] and Van Zyl [17] benchmark networks show the efficiency of our preprocessing, the accuracy of our approximation, and the potential of the overall method to compute near-optimal solutions within seconds where concurrent methods [3, 6, 8, 10, 13] need minutes or hours to compute solutions of higher costs.

The paper is organized as follows: Section 2 defines the problem and describes the standard MINLP formulation in a simplified case. Section 3 presents our non-compact LP formulation and preprocessing reduction techniques and Section 4 the heuristic. Computational results are given in Section 5 and conclusions and perspectives in Section 6.

2 PUMP SCHEDULING PROBLEM

This section describes the problem and a standard formulation in the special case, for the sake of simplicity, of a network equipped with unidirectional pipes, fixed-speed pumps and no valves. Comprehensive formulations for more general networks can be found e.g. in [3, 4].

As illustrated in Figure 1, a water distribution network can be represented as a directed graph $\mathcal{G} = (J, L)$ with sources J_S, junctions J_J and tanks J_T as nodes J, and pipes L_P and pumps K as arcs L. Given a time horizon $\mathcal{T}$ of typically one day, the system dynamics are driven by the water demand rate $D \in \mathbb{R}_+^{J_J \times \mathcal{T}}$ at the junctions and are governed by complex hydraulic laws of conservation of flow and pressure through the network. The problem is to schedule the pump operations over $\mathcal{T}$ in order to continuously satisfy the demand and the allowed filling level of the tanks, while minimizing the operation cost.

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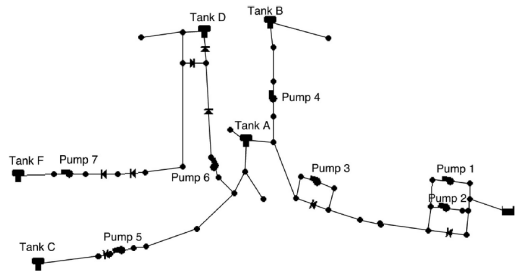


Figure 1: The Poormond network

A standard model is defined as follows, with $x \in \{0,1\}^{K\times\mathcal{T}}$ the binary on/off state of the pumps, $q \in \mathbb{R}_+^{L\times\mathcal{T}}$ the flow rate through the arcs, and $h \in \mathbb{R}_+^{J\times\mathcal{T}}$ the head at the nodes (defined as the sum of pressure and elevation):

$$(P): \min_{x,q,h} \sum_{t\in T}\sum_{k\in K} C_t\Delta_t\Gamma_k(x_{kt}, q_{kt}) \quad (1)$$

$$s.t. \sum_{ij\in L} q_{ijt} = \sum_{ji\in L} q_{jit} + D_{jt}, \qquad t\in\mathcal{T}, j\in J_J \quad (2)$$

$$\sum_{ij\in L} q_{ijt} - \sum_{ji\in L} q_{jit} = \frac{\sigma_j}{\Delta_t}(h_{jt} - h_{jt-1}), \qquad t\in\mathcal{T}, j\in J_T \quad (3)$$

$$h_{j0} = H_j^0, \qquad j\in J_T \quad (4)$$

$$H_{jt}^{\min} \le h_{jt} \le H_{jt}^{\max}, \qquad t\in\mathcal{T}, j\in J \quad (5)$$

$$q_{kt} \le Q_k^{\max} x_{kt}, \qquad t\in\mathcal{T}, k\in K \quad (6)$$

$$h_{it} - h_{jt} = \Phi_{ij}(q_{ijt}), \qquad t\in\mathcal{T}, ij\in L_P \quad (7)$$

$$(h_{jt} - h_{it} - \Psi_{ij}(q_{ijt}))x_{ijt} = 0, \qquad t\in\mathcal{T}, ij\in K. \quad (8)$$

In this model, the time horizon is discretized $\mathcal{T} = \{1,\dots,T\}$ with a resolution Δ_t of typically 1 hour in which the system is assumed to operate in steady state. Constraints (2) and (3) enforce the conservation of flow at junctions and tanks. In (3), a tank $j \in J_T$ is assumed to be a vertical cylinder of area σ_j which links the stored water volume linearly to the head. Bounds on heads (4) and (5) depend on the node types: for a tank $j \in J_T$, they are given by the minimum and maximum filling levels, and by the initial level H_j^0; for a junction $j \in J_J$, $H_{jt}^{\min}$ stands for the minimum pressure required to serve demand D_{jt}; for a source $j \in J_S$, the head is fixed exogenously. Constraints (7) enforce the head losses due to friction in pipes. For each directed pipe $ij \in L_p$, the head loss can be accurately approximated by a quadratic function Φ_{ij} of the flow. Constraints (6) bound the flow through the pumps and bind flow values and pump activation states. Constraints (8) model the head increase through active pumps (if $x_{ijt} = 1$). For each pump $k \in K$, head-flow coupling function Ψ_k can be accurately fitted from operating points as a quadratic function. Finally, the financial cost is mainly incurred by purchasing electricity for pumping, see objective (1) with $C_t \ge 0$ the actualized electricity price on period t, and Γ_k the power consumption of pump k defined by a linear curve fit $\Gamma_k(x,q) = \lambda_k x + \mu_k q$.

Model (P) is a non-convex MINLP which is often untractable even for small networks. One option to solve (P) is to decrease the resolution of the time discretization, and thus the model size, but it has potential drawbacks: (1) the steady-state assumption is less realistic over longer time steps, (2) this artificially reduces the set of feasible schedules, and (3) the optimum increases accordingly. We investigate the opposite option, closer to the reality, by allowing to operate pumps at any time.

3 AN APPROXIMATED NON-COMPACT MODEL

We present a new approximated LP formulation of the pump scheduling problem which separates the computation of the hydraulic balances from the optimization of the schedule. The description is first given in the context of networks where fixed-speed pumps are the only operable elements. We then generalize the definition to networks with valves or variable-speed pumps.

3.1 Tank head approximation

Let $A \subseteq L$ be the set of operable elements of the network, and assume for now that $A = K$ the set of fixed-speed pumps. We call *configuration* any subset $s \subseteq A$, also denoted by its indicator function $I^s \in \{0,1\}^A$ defined by $I_a^s = 1 \Leftrightarrow a \in s$. Configuration s is said *active* at time $t \in \mathcal{T}$ if all its elements, and only these elements, are active (e.g. pumps are on): $x_{at} = 1 \iff a \in s$.

Looking at model (P), a configuration $s \subseteq A$ can be active at time $t \in \mathcal{T}$ only if, given the tank levels and the allowed variation range, the pumps belonging to the configuration offer together enough power to increase head and satisfy the demand rate D_t at all junctions. Because water tanks are usually very large containers, the level variation during 1 hour or less is relatively limited when compared to their heights. We propose to ignore this variation and assume that, at any tank $j \in J_T$, the head is fixed to an arbitrary value H_j^* during time step t. Under this assumption, we can easily compute the hydraulic balance for supplying demand D_t with configuration s. Indeed, by definition, it is a solution $(q_t, h_t) \in \mathbb{R}_+^L \times \mathbb{R}_+^J$ of the non-convex system $\{(2),(5),(6),(7),(8)\}$ restricted to a single time step $\mathcal{T} = \{t\}$ with fixed pump states $x_{at} = I_a^s\ \forall a \in A$ and fixed tank heads $h_{jt} = H_j^*\ \forall j \in J_T$. It is known [5, 6, 16] that this equation system, that we denote $\mathcal{F}_t(I^s, H^*)$, has at most one solution which can quickly be computed, in particular, by the Newton method [16] which is implemented in the popular numerical simulator EPANET [11].

To estimate if a configuration s may supply demand rate D_t, we thus propose to check the feasibility of $\mathcal{F}_t(I^s, H^*)$ with tank heads arbitrarily fixed to their median values $H_j^* = (H_j^{\min} + H_j^{\max})/2$ for all $j \in J_T$. If feasible and (q^s, h^s) its solution, we compute the corresponding instantaneous power consumption $P_t^s = \sum_{k\in s\cap K} \Gamma_k(1, q_{kt}^s)$ and net fill rate $R_{jt}^s = \sum_{ij\in L} q_{ijt}^s - \sum_{ji\in L} q_{jit}^s$ at each tank $j \in J_T$. We denote by $S_t^* = \{s \subseteq A \mid \mathcal{F}_t(I^s, H^*) \neq \emptyset\}$ the set of configurations which may be active during time step $t \in \mathcal{T}$ according to this assumption.

3.2 Configuration scheduling

Given these estimates, we reformulate the pump scheduling problem as a configuration scheduling problem where, at any time step t, any configuration $s \in S_t^*$ is allowed to be active for a duration $0 \le \delta_{st} \le \Delta_t$ within the time step. Modelling the system dynamics boils down to enforce tank head conservation between consecutive time steps, then

leads to the following linear program:

$$(P^*): \min_{\delta,h} \sum_{t\in T} C_t \sum_{s\in S_t^*} P_t^s \delta_{st} \tag{1'}$$

$$s.t. \sum_{s\in S_t^*} \delta_{st} = \Delta_t, \qquad t \in \mathcal{T} \tag{9}$$

$$h_{jt} - h_{jt-1} = \sum_{s\in S_t^*} \frac{R_{jt}^s}{\sigma_j} \delta_{st}, \qquad t \in \mathcal{T}, j \in J_T \tag{3'}$$

$$H_{jt}^{\min} \le h_{jt} \le H_{jt}^{\max}, \qquad t \in \mathcal{T}, j \in J. \tag{5'}$$

In (P^*), pumps can thus be operated during time steps. Furthermore, (P^*) is an approximation of (P), and not just a relaxation: an optimal configuration s at time t for (P) may not belong to S_t^* if it can indeed satisfy demand D_t but not under the half-filled tanks assumption; if s does belong to S_t^*, otherwise, then its actual consumption may not be P_t^s precisely.

3.3 Processing configurations

Computing one hydraulic balance with the Newton algorithm is almost immediate, but the exponential number of configurations to evaluate, in $O(T2^{|A|})$, can make it computationally challenging to build (P^*). We propose two techniques to significantly reduce the computation time without hindering optimality.

First, we exploit the symmetries, which are frequent in real data. For instance, when demand rate $D_t \in \mathbb{R}^{J_J}$ is constant at all junctions over a number of time steps, then the configurations need to be evaluated on only one time step, since if $D_t = D_{t'}$, then $S_t^* = S_{t'}^*$, $P_t^s = P_{t'}^s$ and $R_t^s = R_{t'}^s$ for all $s \in S_t^*$. Another symmetry arises when pumps with identical characteristics are installed in parallel at a pumping station (see e.g. pumps 1 and 2 in Figure 1). Only one symmetric configuration is then evaluated.

Second, we exploit a partition of the network along the tank nodes. Precisely, we consider the graph $\mathcal{G}' = (J', L')$ obtained from $\mathcal{G}$ by duplicating each tank node $j \in J_T$ for each incoming arc $ij \in L$ as a new node denoted j^i, i.e. $J' = J \cup \{j^i \mid ij \in L, j \in J_T\}$ and $L' = L \cup \{ij^i \mid ij \in L, j \in J_T\} \setminus \{ij \in L \mid j \in J_T\}$. Once the heads at tanks $j \in J_T$ (and at duplicate nodes j^i) are fixed to their median values H_j^*, the arc flow and node head values become independent in each connected component of $\mathcal{G}'$. We thus compute the set of feasible configurations $S_t^C \subseteq A \cap L_C$ independently for each connected component $C = (J_C, L_C) \in \mathcal{CC}(\mathcal{G}')$. These sub-configurations are then combined by summation: for all $s \subseteq A$, $P_t^s = \sum_{C\in\mathcal{CC}(\mathcal{G}')} P_t^{s\cap L_C}$ and $R_{jt}^s = \sum_{C\in\mathcal{CC}(\mathcal{G}') \mid j\in J_C} R_{jt}^{s\cap L_C}$ for $j \in J_T$. Hence, the network partition reduces both the number of computations and their complexity, being evaluated on smaller graphs. Furthermore, the symmetry condition on constant demand occurs with a higher frequency when regarding the subsets of junctions independently.

3.4 Generalization

In most water distribution networks, not only pumps but also valves $V \subseteq L$ of different types can be operated. Like fixed-speed pumps, gate valves and check valves have only two possible states (close or open) and can be modeled in (P) with a binary variable for each time step (see e.g. [4]). The definition of configuration can then be extended to $A = K \cup V$ the set of pumps and valves, saying that a valve is active if it is close. Furthermore, according to [12], $\mathcal{F}_t(I^s, H^*)$ has still at most one solution.

The presence of variable-speed pumps or pressure-reducing valves deserves more attention as they admit a continuous range of operation modes. A variable-speed pump is either off or operated within an allowed range of speed. For a pressure-reducing valve, the amount of pressure reduction is chosen within a given range and a binary state indicates the direction of the flow (see e.g. [15]). As suggested in [5], we propose to approximate the allowed operation range of each pressure-reducing valve or variable-speed pump a by a discrete set of sample values A_a. The set A is then augmented with these sample values and a configuration is now defined as $s \subseteq A$ with $|s \cap A_a| \le 1$. Once pump speeds and pressure reductions are fixed in configuration s, the Newton method can quickly solve $\mathcal{F}_t(I^s, H^*)$ as before.

Hence, the approximation model (P^*) and configuration processing scheme apply to a comprehensive class of water networks. Still, the number of configurations to evaluate grows exponentially with the number of operable elements, unless the network partition separates these elements in small sets so that the growth becomes near linear.

4 BENDERS DECOMPOSITION-BASED HEURISTIC

This section describes an adaptation of the combinatorial Benders decomposition of [10] to search, in the neighborhood of the approximated solutions of (P^*), feasible solutions to the pump scheduling problem with pump aging constraints.

4.1 Pump aging

While in practice pumps can be operated at any time, too frequent switches are prohibited to prevent premature pump aging. Ghaddar et al. [6] proposed to enforce the following constraints in model (P) for each pump $k \in K$:

$$\sum_{t\in\mathcal{T}} y_{kt} \le N, \tag{10}$$

$$y_{kt} \ge x_{kt} - x_{k(t-1)}, \qquad t \in \mathcal{T} \tag{11}$$

$$x_{kt'} \ge y_{kt}, \qquad t \in \mathcal{T}, t' \in [t, t+\tau_1] \tag{12}$$

$$z_{kt} \ge x_{k(t-1)} - x_{kt}, \qquad t \in \mathcal{T} \tag{13}$$

$$x_{kt'} \le 1 - z_{kt}, \qquad t \in \mathcal{T}, t' \in [t, t+\tau_0] \tag{14}$$

with y_{kt} (resp. z_{kt}) a binary variable equal to 1 if pump $k \in K$ is switched on (resp. off) at time t, N the maximal number of times a pump can be switched on, τ_1 (resp. τ_0) the minimum continuous duration a pump is on (resp. off).

Naoum-Sawaya et al. [10] designed a combinatorial Benders decomposition approach, where the master integer linear program denoted (M) is initialized with the aging constraints (10)-(14) alone. At each iteration, (M) returns a candidate schedule $X \in \{0,1\}^{K\times\mathcal{T}}$ to evaluate: the EPANET simulator computes the hydraulic balance and power consumption at each time step, sequentially. If a hydraulic constraint is violated or if the partial cost exceeds the best solution known so far at a given time $\bar{t} \in \mathcal{T}$, then the partial schedule up to time $\bar{t}$ is discarded from the

search by adding a no-good cut to master (M):

$$\sum_{t=1}^{\bar{t}} \left(\sum_{\substack{k \in K \\ X_{kt}=0}} x_{kt} + \sum_{\substack{k \in K \\ X_{kt}=1}} (1 - x_{kt}) \right) \geq 1. \qquad (15)$$

The cut is also added with $\bar{t} = T$ each time X proves to be the new incumbent, i.e. an improving solution. The algorithm stops when (M) becomes unfeasible. The algorithm theoretically converges to a certified optimal schedule, but its slow convergence requires to limit the computation time.

4.2 Truncated Benders decomposition

In the original method [10], the objective function of master (M) is initialized to 0, then systematically redefined as the minimal distance to the new incumbent, so as to search the next candidate in a neighborhood. Thus, the algorithm improves the solution progressively, as in a local search, but it may start with a low quality solution. We propose to adapt this algorithm to conduct it explicitly as a heuristic to compute good solutions fast. To this end, we initialize the algorithm with an optimal approximate solution δ^* of (P^*), then adjust the distance function, i.e. the neighborhood, iteratively until finding a feasible solution.

More precisely, we first compute the duration $\delta^*_{at} = \sum_{s \in S^*_t} I^s_a \delta^*_{st}$ of activation of any pump or valve $a \in A$ during time step $t \in \mathcal{T}$ in the approximated solution. We expect that, in an optimal schedule x, if δ^*_{at} is close to Δ_t then a is active during t (i.e. $x_{at} = 1$), and that, if δ^*_{at} is close to 0 then a is inactive during t (i.e. $x_{at} = 0$). Furthermore, we estimate that the daily duration of activation $\sum_{t \in \mathcal{T}} x_{at} \Delta_t$ of a is close to $\sum_{t \in \mathcal{T}} \delta^*_{at}$. Hence, the minimization criterion of (M) is initialized to:

$$\sum_{a \in A} \sum_{t \in \mathcal{T}} (\Delta_{at} - x_{at}\Delta_t)^2 + \sum_{a \in A} (\sum_{t \in \mathcal{T}} \Delta_{at} - \sum_{t \in \mathcal{T}} x_{at}\Delta_t)^2 \quad (16)$$

with $\Delta_{at} = \begin{cases} \delta^*_{at} & \text{if } \delta^*_{at} \in \{0, \Delta_t\} \\ \alpha_{at} & \text{otherwise,} \end{cases}$ and $\alpha_{at} \in \{0, \delta^*_{at}, \Delta_t\}$ is a parameter of diversification which is initialized to δ^*_{at} to search first around the approximated solution of (P^*), and is then updated randomly at each iteration.

Another difference with [10], is that we generalize the method to networks with variable-speed pumps or pressure-reducing valves. In this context, we propose to use a non-convex NLP solver instead of the EPANET simulator to evaluate the candidate solutions $X \in \{0,1\}^{A \times T}$ by solving the slave program, i.e. the standard MINLP formulation with the binary variables x fixed to values X. Note here that, unlike for the processing of the configurations (see Section 3.4), we do not extend the definition of the set of operable elements A by discretizing the continuous state range for variable-speed pumps and pressure-reducing valves. Finally, because we run the Benders decomposition as a heuristic, the slave problem is not required to be solved at optimality. Hence, when the global optimization of the restricted non-convex NLP is too time consuming, a fast local optimization solver can be used instead.

5 COMPUTATIONAL RESULTS

We experimented the full heuristic, sketched in Algorithm 1, on two benchmark sets: Poormond [6] and Van Zyl [17]. In this section, we evaluate the solutions in comparison with the best solutions known so far for these instances (see [3] for a comparative analysis of the results published in [3, 6, 10, 13] on Poormond).

Algorithm 1: Heuristic for (P)

```
for C ∈ CC(G'), s ⊆ A ∩ L_C, t ∈ T do
    solve F(I^s, H*) with Newton method
compute P^s_t, R^s_t by summation ∀s ∈ S*_t, t ∈ T
solve LP (P*): get δ*
initialize (M): min (16) s.t. (10)-(14)
while unfeasible do
    solve MIQP (M): get X
    simulate X with Newton method or NLP solver
    if X feasible then
        return X
    else
        add cut (15) to (M)
        update (16)
```

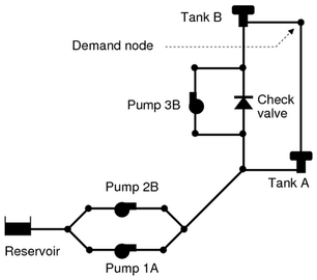


Figure 2: The Van Zyl network

5.1 Experimental set-up

The Poormond network, depicted in Figure 1, was derived by [6] from the real water distribution network of Richmond, England. It is a medium-size network with 47 nodes including 1 source and 5 tanks, 44 pipes, 7 fixed-speed pumps and 4 gate valves. The benchmark set has five daily instances, denoted from P21 to P25, each corresponding to the real dynamic power tariff, available at [14], that occurred each day in range May 21-25, 2013. The time horizon is discretized in $T = 48$ time steps of $\Delta_t = 1/2$ hour each. Pumps are required to stay on for at least 1 hour ($\tau_1 = 2$), off for at least 1/2 hour ($\tau_0 = 1$), and to be activated at most $N = 6$ times. The Van Zyl network [17], depicted in Figure 2, is a fictive, small but complex network with 1 source, 2 tanks, 15 pipes, 1 check valve and 3 pumps assumed to be variable-speed pumps after [8]. We experimented on this network using the same 5 tariff profiles set at the same time resolution. We denote the five instances Z21 to Z25 accordingly.

The computations were performed on a Xeon E5-2650V4 2.2GHz with 254 GB RAM. The processing of the configurations, including the Newton method, was implemented in Python, while the default LP solver and MINLP solver of Gurobi 7.0.2 were run on one thread to solve (P^*) and (M) respectively. For the Van Zyl instances, the slave problems of the Benders decomposition were solved with the default non-convex NLP local solver of Bonmin [1]. The step to

discretize the allowed pump speed range was empirically fixed to $|A_a| = 6$.

The heuristic solutions are compared with the best solutions returned by the branch-and-cut method of [3] running in 1 hour under the same experimental set-up. In [3], a MILP outer approximation of (P) is solved with a LP-branch and bound augmented with user cuts: at each integer node, the corresponding pump configuration is evaluated as in the Benders decomposition here described, and no-good cuts generated accordingly. To make the comparison of the results valid, we implemented the exact evaluation procedure described in [3]. It includes, for the Poormond instances, a primal heuristic which slightly adjusts the duration of activation of the pumps to correct the small bound violations induced by the fixed time discretization.

5.2 Quality of the approximation

Figure 3 illustrates on instance P21, for each of the 7 pumps of the Poormond network, the optimal pump schedule δ^* returned by (P^*) after recomputation of the real flows and heads (in blue), and the feasible pump schedule returned by our heuristic (in pink). Figure 4 depicts the water filling profiles of the 5 tanks for both solutions and, below, the dynamic electricity tariff profile.

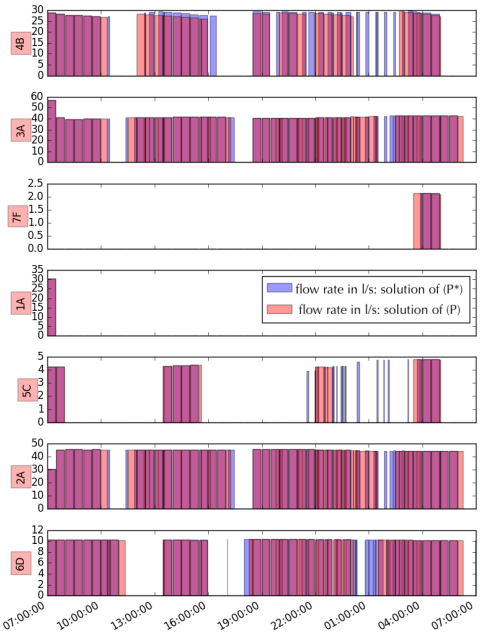


Figure 3: Approximated and feasible schedules

We observe for the approximated solution on Figure 4 that the water levels in the tanks (the blue curves) only slightly fall outside the allowed range (delimited by the black lines) which indicates that the approximated pump schedule is close to be practically feasible. When considering the modelling errors and the security margins and ignoring the formal pump aging constraints, this approximated schedule could probably directly be applied as a command for the real-time control of the pumps.

The near feasibility of the solution attests the relevancy of approximating the tank heads to their median values.

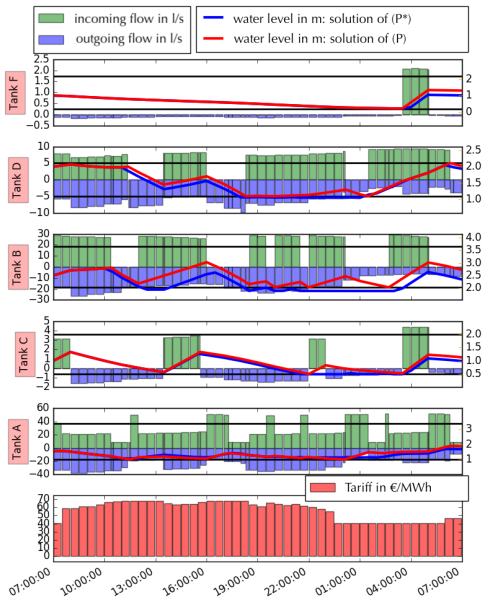


Figure 4: Tank levels in the approximated and feasible solutions

Indeed, we observe an average relative deviation lower than 1% between the flow profiles delivered by the pumps, before and after recomputation with the actual tank heads. This confirms our hypothesis, we observed on a sample configuration, that the error on the flow due to this approximation is significant only when some tanks are empty while others are full. Here, on the contrary and as expected, the filling profiles of the 5 tanks all follow the same dynamic generated by the variable electricity tariff.

Perhaps more surprising, we observe on Figure 3 that the approximated and feasible pump schedules overlap extensively, from 77% for pump 5C to 100% for pump 1A, which indicates that the approximated solution mostly satisfies the fixed time discretization constraint of model (P) and the pump aging constraints, although they are entirely relaxed in (P^*). Actually, because (P^*) has comparatively few constraints $(O(T|J|))$, a basic solution has then few columns. In other words, only a fraction of the configurations over all the time steps have a non-zero duration in the optimal approximated schedule. For instance P21 depicted here, only 104 configurations are active which corresponds to 3% of the generated configurations, and, on the 48 time steps, 15 are associated to an unique configuration. This explains why pumps are activated at reasonable frequency, from 1 for pump 1A to 21 for pump 4B, in the approximated solution.

Finally as the approximated and feasible solutions are close, their costs (111.03 euros for the former and 117.50 for the latter) present a moderate gap (+6%). We observed the same proximity on all the Poormond instances and on all the Van Zyl instances too. For example, in instance Z21, the approximated solution has only 50 active configurations on more than 20,000 candidates over the 48 times steps and it satisfies all the pump aging constraints. Only one iteration of the Benders decomposition and a slight adjustment of

	Computation time (s)				Cost (euros)			
Day	conf	LP	TBD	Total	TBD	best[3]	TBD/LB	best/LB
P21	1.6	<0.1	16.3	17.9	117.50	112.48	8.2%	4.1%
P22	1.6	<0.1	11.2	12.8	118.55	116.49	5.6%	3.9%
P23	1.6	<0.1	8.0	9.6	120.93	120.85	4.1%	4.0%
P24	1.6	<0.1	10.9	12.5	137.05	134.99	4.6%	3.1%
P25	1.6	<0.1	21.2	22.8	98.74	92.53	9.8%	3.8%
Z21	2.1	<0.1	0.7	2.8	220.60	222.66	14.9%	15.7%
Z22	2.1	<0.1	1.7	3.8	230.07	230.69	14.1%	14.3%
Z23	2.1	<0.1	1.4	3.5	240.67	240.93	13.7%	13.8%
Z24	2.1	<0.1	0.6	2.6	267.77	268.91	14.4%	14.7%
Z25	2.1	<0.1	0.7	2.8	188.52	190.29	14.5%	15.3%

Table 1: Results of the heuristic

the pump speeds were needed to retrieve a feasible solution with a +4.2% cost deviation.

5.3 Performance of the heuristic

Table 1 summarizes the computational results of our heuristic on the 10 instances of Poormond and Van Zyl. On the left part, the computation times (in seconds) are detailed for each algorithmic component: the preprocessing of the configurations (conf), the solution of the approximated model (P^*) (LP), and the truncated Benders decomposition (TBD). The right part of the table gives the costs (in euros) of the solutions returned by our heuristic (TBD) compared to the solutions of the branch-and-cut approach returned in 1 hour (best [3]); TBD/LB and best/LB denote the respective optimality gaps to the best know lower bound also returned by the branch-and-cut in 1 hour.

We observe that the heuristic computed good quality solutions fast. About 2 seconds were required to generate an approximated schedule, mostly to preprocess the set of configurations since solving the LP was immediate. The graph partition has a great impact on the number of configurations to process. On the Van Zyl network, for example, the partition creates two components: the one with all the operable elements but no demand – resulting in 456 configurations which are identical for each time step (even for each instance, actually) – and the other with the unique demand node, only two pipes and no operable elements – resulting in one configuration for each time step. Hence, we computed $456 + 48$ hydraulic balances instead of 456×48.

The truncated Benders decomposition ran in 14 seconds in average on Poormond and in 1 second on Van Zyl. It stopped with a feasible solution after the first iteration, except for instance P25 which required two iterations. On Poormond, the costs of the heuristic solutions were, in average, 2.9% higher than the best solutions, and up to 5% higher for instance P25. In comparison, the branch-and-cut required 306 seconds in average to compute a first feasible solution of comparable quality (at 2.6% of the final solutions). According to [3], our heuristic solutions also improve upon the solutions reported by [6] and [10] after 1 hour of computation. On Van Zyl, the heuristic computed in 3 seconds, in average, solutions which slightly improve upon the solutions found in 1 hour by the branch-and-cut. The average optimality gap is 14.3%. In comparison, [8] reported approximated solutions with a 30% optimality gap computed in 5 minutes by solving a MILP obtained by piecewise linearization of the non-convex constraints in (P).

6 CONCLUSION

We formulated the pump scheduling problem in water distribution network as a new generic non-compact linear program, based on the approximation of the head at the water tanks and on the relaxation of the pump aging constraints. This approximation turned out to be both tight and easy to solve when experimented on two networks with different characteristics. We were then able to quickly find low cost feasible solutions by searching in a neighborhood of the approximated solutions. These results lead us to believe that this method could deal with networks larger than with the currently known approaches. Failing to dispose of such study cases, we envisage to build new realistic instances to confirm our claim. Perspectives to extend our method are, first, to exploit the new LP approximation in a global optimization approach, and, second, to exploit historical data of network operations to build the configuration set.

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