Investigating a Hybrid Approach for Global Optimization of Pump Scheduling Problem

A. Tavakoli, V. Sessa, S. Demassey

Centre for Applied Mathematics, MINES ParisTech, PSL Research University, Sophia Antipolis, France

AI for Smart and Secure Territories

Abstract

Pump scheduling is a decision-making problem in water distribution networks. The aim is to plan the pumping operations to minimize the energy cost over the day ahead. Modelling the binary status of the pumps and the nonconvex pressure-flow relations throughout the network results in non-convex Mixed Integer Non-Linear programs (MINLP) that could be particularly hard to solve. The branch-and-check algorithm [1] implemented on top of a commercial linear solver to guarantee the global optimization paradigm for solving such non-convex MINLPs is viable due to convexification of malign constraints. The looseness of convexifications (relaxations) exacerbates the convergence of the optimization process. In response to these caveats, we propose bound tightening and generation of valid inequalities (i.e., cutting planes) at preprocessing stage. This may mitigate the effect of relaxations in form of continuous or nonlinearity, yet potentially too costly. We have proposed a surrogate model to efficiently lower estimate some bounds and a heuristic to control the generation of such cuts.

1. Pump scheduling

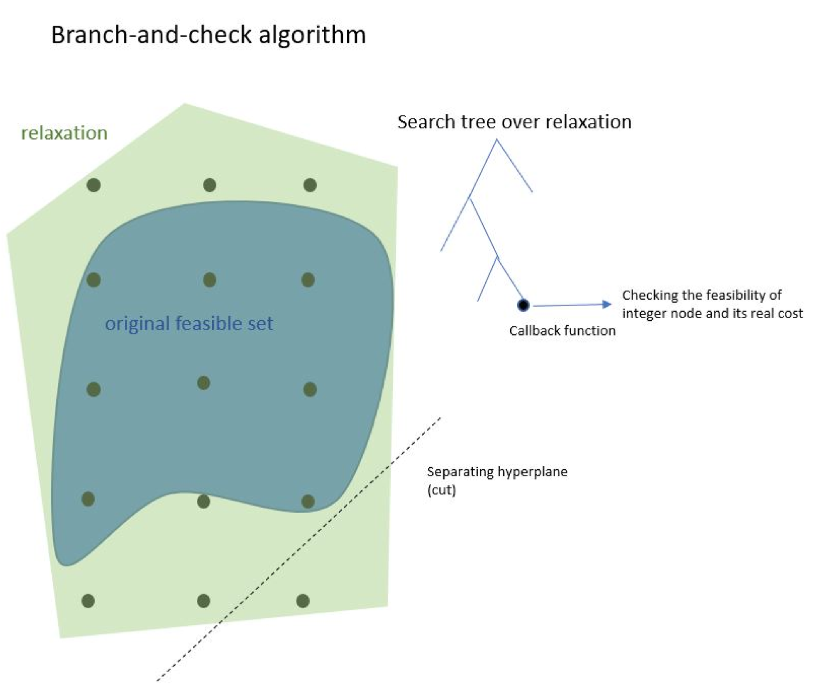
Scheduling the status of pumps at each time step ($x_{kt} = \{0,1\}\ \forall k \in K\ \ \forall t \in \mathbb{T}$) in a way that the tariff incentive energy bill is minimized while the given demand profile and physical constraints are met.
The caveats of such a problem are:

- **integrality** of decision variables
- non-linear equality constraint in pumps and pipes' pressure-flow relationship enforcing **non-convexity**

As a result, a large-scale non-convex MINLP intractable for global solvers

2. Branch-and-check algorithm

Relaxation of the non-convex pressure-flow relationship via outerapproximation ($\mathcal{R}^{OA}$)[1]. Linearized version of the feasible set is larger than original non-convex set. This requires checking the feasibility and cost of each encountered integer node in branch and bound approach.



3. Strengthening mathematical formulation at the preprocessing stage

- **Optimization-based Bound tightening, OBBT**[2]: defining auxiliary optimization problem, the max/min of each variable leading to a feasible solution over relaxed version of the original constraints [3]
- **Cutting planes**: separating infeasible solutions from the relaxed polyhedral set
  - tackling the non-linearity by **shaving**, **disjunctive programming**, and **strong duality**[4]
  - tackling the integral complexity
    - **flow cover inequalities** by *lifting* via Mixed-integer rounding and superadditivity
    - **cardinality cuts**: exploiting underlying structure of sceduling problems; minimum number of required ON pumps until a certain time step (e.g, $t' \in \mathbb{T}$) by decomposition, can be too much time consuming

4. Experimental result

Experimental result over a benchmark, C0: no preprocessing, C1: with preprocessing

Benchmark	Formulation	instance	ub	lb	gap	1st
Richmond 12	C0	1	NA	110.7	NA%	NA
		2	NA	113.5	NA%	NA
		3	NA	123.93	NA%	NA
		4	NA	137.19	NA%	NA
		5	NA	112.48	NA%	NA
Richmond 12	C1	1	114.09	114.09	0%(757s)	40.6
		2	117.54	117.54	0%(2088s)	67.6
		3	130.3	130.3	0%(3982s)	211.8
		4	141.57	141.57	0%(916s)	180.6
		5	117.08	117.08	0%(938s)	45.3
Richmond 24	C0	1	113.2	107.80	4.8%	572
		2	114.41	110.6	3.2%	125.3
		3	126.0	121.5	3.6%	319
		4	140	134.60	4.5%	2001
		5	96.1	92.8	5.8%	450
Richmond 24	C1	1	111.04	108.32	2.4%	48.6
		2	113.77	111.27	2.2%	159.9
		3	125.28	122.43	2.3%	74
		4	138.5	134.91	2.6%	44.6
		5	96.07	94.13	2%	204.5

ub: upper bound, lb= dual bound, gap: optimality gap between ub and lb (time to proof optimality), 1st: the time to obtain first feasible solution

5. Surrogate model for cut generation

The cuts should:

- have a tractable generation process
- be tight enough to remove infeasible regions as much as possible

These two criteria are usually orthogonal
conditional cardinality cuts: finding the lower estimation of required number of active variables for a decomposed problem under some conditions.

- **heuristic** to find appropriate conditions
- **surrogate model** to lower estimate the RHS of inequalities based on these conditions

Surrogate model is viable by aggregating subsets of constraints and nodes resulting to a less complex graph consisting of supernodes and cutsets

6. Heuristic for cut generation

Algorithm 1 Heuristic for generations of conditional cardinality cuts

Input lower and upper bounds of variables $\{\underline{\zeta}, \overline{\zeta}\}$, Relaxation R^{OA}, $K_1 \subset K$, $K_2 \subset K$ subsets of pumps, δ the accuracy of head-flow relaxation

Output valid inequalities according to some scenari

1: $\mathcal{R}^M \leftarrow \Pi(\mathcal{R}, \delta, \{\underline{\zeta}, \overline{\zeta}\})$create a surrogate model based on relaxation $\mathcal{R}$ and accuracy tolerance δ. call the surrogate relaxation set as $R^{\mathcal{M}}$

2: $\bar{Y} \leftarrow \{\min \mathcal{P} | \bar{\mathcal{R}}^{OA}\}$solving continuous LP relaxation of the scheduling problem $\mathcal{P}$ over $\mathcal{R}^{OA}$ and obtain the solution $\bar{Y}$

3: sort time steps according to highest electricity tariff and separate l highest $T^l \subset T$. compute the summation of the relaxed activities of pumps ($\bar{x}_{at} \in [0,1] \subset \bar{Y}$) during these time intervals $\bar{X} = \sum_{t \in T^l} \sum_{a \in K_2} \bar{x}_{at}$

4: solve the lower estimation of required number of active pumps to time step t' with

- † $\underline{X}_0^{[0,t']} \leftarrow \min\{\sum_{t=0}^{t'} \sum_{ij \in K_1} | \mathcal{R}^{\mathcal{M}} \wedge \sum_{ij \in K_2} \sum_{t \in T^l} x_{ijt} \le \lceil \bar{X} \rceil\}$
- † $\underline{X}_1^{[0,t']} \leftarrow \min\{\sum_{t=0}^{t'} \sum_{ij \in K_1} | \mathcal{R}^{\mathcal{M}} \wedge \sum_{ij \in K_2} \sum_{t \in T^l} x_{ijt} \le \lceil \bar{X} \rceil + 1\}$
- † $\underline{X}_2^{[0,t']} \leftarrow \min\{\sum_{t=0}^{t'} \sum_{ij \in K_1} | \mathcal{R}^{\mathcal{M}} \wedge \sum_{ij \in K_2} \sum_{t \in T^l} x_{ijt} \le \lceil \bar{X} \rceil + 2\}$

5: adding conditional constraints such as $\sum_{t=0}^{t'} \sum_{ij \in K_1} x_{ijt} \le (\sum_{ij \in K_2} \sum_{t \in T^l} x_{ijt} \le \lceil \bar{X} \rceil \implies \underline{X}_0^{[0,t']}) \vee (\sum_{ij \in K_2} \sum_{t \in T^l} x_{ijt} \le \lceil \bar{X} \rceil + 1 \implies \underline{X}_1^{[0,t']}) \vee (\sum_{ij \in K_2} \sum_{t \in T^l} x_{ijt} \le \lceil \bar{X} \rceil + 2 \implies \underline{X}_2^{[0,t']})$ to $\mathcal{R}^{OA}$

6: linearizing constraints by big-$\mathcal{M}$ formulation

Conclusions

- Strengthening mathematical formulation of pump scheduling problem is inevitable due to relaxations
- Bound tightening and cutting generation are promising methods to speed up the convergence of optimization process in the framework of global optimization
- Some preprocessing techniques demands high computational cost, we address this issue by introducing a surrogate model and a heuristic
- This heuristic can be superceded by a learning algorithm

References

[1] Gratien Bonvin, Sophie Demassey, and Andrea Lodi. Pump scheduling in drinking water distribution networks with an lp/nlp-based branch and bound. *Optimization and Engineering*, pages 1–39, 2021.

[2] Ambros M Gleixner, Timo Berthold, Benjamin Müller, and Stefan Weltge. Three enhancements for optimization-based bound tightening. *Journal of Global Optimization*, 67(4):731–757, 2017.

[3] Sophie Demassey, Valentina Sessa, and Amirhossein Tavakoli. Strengthening mathematical models for pump scheduling in water distribution. *Energy Procedia*, 2022.

[4] Sophie Demassey. Enhanced branch & check for pump scheduling in water networks. In *MINLP Workshop Mixed-integer nonlinear optimisation 2021*, 2021.